THE UNIVERSITY OF ALBERTA

THE APPLICATION OF THE THEORY OF STOCHASTIC PROCESSES TO PRECIPITATION AT SOME ALBERTA STATIONS

by

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research for acceptance, a thesis entitled "THE APPLICATION OF THE THEORY OF STOCHASTIC PROCESSES TO PRECIPITATION AT SOME ALBERTA STATIONS", submitted by William Meheriuk in partial fulfilment of the requirements for the degree of Master of Science.

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research for acceptance, a thesis entitled "THE APPLICATION OF THE METHOD OF SYNOPTIC FORECASTING TO PRECIPITATION AT SOME ALBERTA STATIONS", submitted by William [illegible] in partial fulfilment of the requirements for the degree of Master of Science.

ABSTRACT

The stochastic model of the precipitation regime developed by Verschuren and Todorovic can be applied to particular climatological stations or geographic areas if two parameters can be estimated from past records. These parameters, λ_1 and λ_2, describe the temporal distribution of the number of storms and of the total amount of precipitation respectively. The present thesis represents an attempt to evaluate these parameters and their variability through the year for thirteen climatological stations in Alberta.

Eleven stations were selected to represent two east-west traverses across Alberta at the latitudes of Lethbridge and Edmonton approximately, and two further stations in southern Alberta were included to evaluate the effects of elevation. While first-order stations were desired, their sparse distribution necessitated the use of eight ordinary climatological stations. As these latter stations report on a daily basis only, the study was restricted to daily time intervals. From twenty-five to thirty-one years of record were available at each station.

The theoretical development of Verschuren and Todorovic is summarized briefly with emphasis on the basic assumptions. Estimating equations for the parameters λ_1 and λ_2 are derived and the variability of the parameters (characteristic curves) is established by obtaining estimates of the two parameters for 71 five-day periods through the year.

Comparison of the characteristic curves for the thirteen stations indicates that significant differences in the distribution of

of precipitation occur. These differences appear to be partly the result of geographic location and elevation and partly due to different operating procedures between first-order and ordinary climatological stations. The minimum number of years of record required to provide reasonable estimates of the parameters λ_1 and λ_2 is also established.

Finally, the effectiveness of the theoretical equations is illustrated by comparing the observed distribution of the number of days with measureable precipitation and of the amount of precipitation for varying time intervals with computed values based on the estimates of λ_1 and λ_2. The reasonable agreement obtained for most stations indicates that the theoretical model is a reasonably close representation of the precipitation process. The model provides useful estimates of the two main variables, which are the number of days with measureable precipitation and the total amount of precipitation in any designated time period.

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LIST OF SYMBOLS

Definition

SYMBOL	
η_t	Number of storms during time interval (t_o,t)
ν	Integer used to count storms
X_ν	Total precipitation during ν storms
X_t	Total precipitation during the time interval (t_o,t)
T_x	Time for an amount of precipitation equal to x to have fallen
t	Time
x	Amount of precipitation
λ_1	Characteristic parameter
λ_2	Characteristic parameter
ξ_t	Precipitation intensity at time t
Z_ν	Total precipitation during the νth storm
t_o	Time of beginning of observations
x_o	Total precipitation at time t_o
ζ_t	Amount of precipitation during a single storm at time t
$F(x/t)$	Distribution function of the random variable η_t
$F_t(x)$	Distribution function of the random variable X_t
ω	Elementary event

CHAPTER I

INTRODUCTION

1.1 Outline of the Project

The main theme of this project is to study the precipitation patterns at various locations in Alberta and compare their distributions with those distributions that have been theoretically derived for various variables associated with the precipitation phenomenon. In general, this study involves:

1. The development of characteristic parameters for the station or area under consideration. These parameters will indicate seasonal changes and therefore, will have to be considered as time-dependent. With the aid of these parameters, the distribution functions for several variables can be calculated.

2. The comparison of the parameters for an ordinary climatological station and that of a nearby first-order station.

3. The comparison of the characteristic parameters for different geographic locations in Alberta.

4. The minimum number of years of record to give reasonable estimates of the characteristic parameters.

1.2 Reasons and Aims

The full potential of the water resource for a specific area or region can only be realized when its magnitude is known. From a

hydrologist's point of view, often there is insufficient information about the flows in many rivers because of the inaccessibility factor or the insufficient instrumentation of such rivers. A commonly used technique in estimating river flow is that of the precipitation - runoff relationship (Chow, 1964; Linsley and Franzini, 1964; Foster, 1949) where the unit hydrograph or other systems are employed. In hydrograph analysis, the excess precipitation is converted to runoff. Before explicit accuracy in determining runoff can be obtained, more emphasis should be directed to the understanding of the occurrence of precipitation.

The seasonal variations of precipitation become important when one considers the loss of life and property damage that may be possible in the event of a flood or the critical effect of a low water supply when a hydraulic structure such as a dam is built across a river. The reservoir of any hydraulic structure should be designed to accommodate a large storage for the high rate of runoff during spring as well as provide ample supply to large drawdown demands during the summer when precipitation is less. Computations requiring the determination of the amounts of precipitation depend on many factors of which many in turn are dependent on the season.

A common practice in designing the safety margin of a dam (Chow, 1964; Linsley and Franzini, 1964) is that of transposing the worst storm that occurred in a particular area during a certain season to a region that is usually described as climatologically similar. If such maximization and transposition of storms is to be allowed, the difference and variation of the climate in the two regions with respect

to the season of the year must be carefully examined in order to avoid improper estimation of the runoff produced by the storm. To reduce the complications in developing the design flood through the method of transposition of storms, a technique adequately describing a flood through the distribution classification of precipitation can be used as an alternative.

Since no event of the precipitation phenomenon can be predicted with certainty, only probabilities of the future outcomes can be determined on the basis of the present state. Therefore, those distribution functions that are dependent on the season of the year will be most applicable to give a description of the variables associated with precipitation.

For any particular season of the year, if one considers the number of storms or the total amount of precipitation in a specified time interval (variables associated with the distribution of precipitation), it has been shown (Verschuren, 1968; Todorovic, 1967) that distribution functions that are theoretically developed and which make use of the characteristic parameters for a station or area can be of assistance when precipitation is considered in the engineering design of hydraulic structures. It has been suggested by various authors (Verschuren, 1968; Thom,1968; Cramer and Leadbeater, 1967) that theoretically derived distribution functions allow a better understanding of the conditions for which these distributions hold and offer a better chance for explanation and correction of differences between theory and the distributions obtained from the observed data than do distribution functions obtained by experimentally fitting the observed

data to well known distribution functions.

The goal of any weather modification program (U.S. Dept. of the Interior, 1970) is to learn how to manage precipitation in water deficient areas by cloud-seeding and to do it in an efficient, economic and socially acceptable manner. Most programs have been developed with the aim of increasing precipitation, especially snowfall in mountainous areas, by the cloud-seeding of mainly orographic weather systems. Before any programs can be accomplished effectively, an extensive statistical study must be undertaken to develop test and control areas so that precipitation changes can be accurately determined. In order that proper evaluation can be made of the precipitation at the ground surface, several parameters are required to describe the distribution of the various variables associated with the precipitation phenomenon.

1.3 Importance of Precipitation in Hydrology

The emphasis on precipitation in hydrology has arisen because records can be obtained easily and cheaply. In many regions, records have been maintained for long periods and over extensive areas. In some countries they are the only available record. Precipitation depths can be converted logically to stream flows and hydrographs. By appropriate methods, precipitation can also be extrapolated rationally to an extreme or limiting value. For example, the probable maximum precipitation has many uses in flood estimation. Statistical methods are readily applicable to precipitation data.

The greatest need for statistical treatment of precipitation

frequencies, intensities and distribution is felt in various economic or related studies. The capacities of sewerage and drainage works are usually designed on the basis of a certain depth of rainfall to be expected during a specified period of time. Flood control works are frequently designed according to a similar criteria, particularly if loss of human life is not probable and the protective works are proposed primarily as insurance against property loss.

1.4 Difficulties

When this project was initiated, it was desired to study the precipitation distribution in mountainous areas and develop characteristic curves that would be representative of the watersheds of some of the major rivers in Alberta. There are many stations in the vicinity of the mountains that report precipitation but unfortunately many of these are forestry lookout towers and ranger stations with records generally ten years or less in duration. Also, a majority of these stations are functional in the summer months only.

At least 30 years of continuous records were chosen for this study and the stations selected are operated by the Atmospheric Environmental Service of Canada. The only first-order stations in the mountainous regions that provide any length of continuous records are those of Jasper and Banff. These are inside of the mountain range and are considered not to be representative of the river basins that are located on the lee side of the mountains. The other stations providing precipitation data in the vicinity of the mountains are classed as ordinary climatological stations and are usually operated by volunteer

observers. A description of the different types of climatological stations will be given in a later chapter.

While many stations contained the desired 30 years of precipitation records, the problem was the fact that the records for some stations were not continuous. It was common to find that records were missing completely for whole months and in some instances the record was missing for the entire year. Another important problem was due to the manner in which the precipitation was recorded. For example, if the observer noted that it rained on day 1 but did not measure the amount of rain until day 2, he would enter a 'C' on the form for day 1 and the precipitation total would be entered for day 2 and thus would represent a two-day rainfall, if in fact, it rained on day 2. Since it rained on day 1, it was entered as a day with rain. However, it is impossible to tell from the records whether or not it rained on day 2, but as the observer measured the total rain on day 2, it had to be entered as a day with rain also.

The same type of problem occurs when the observer is absent from his station for more than one day. When he arrives from his absence and has a measureable amount of precipitation in the gauge, this amount is entered in the records on the day of his arrival and is counted as a day with rain regardless of whether or not it rained at that time. For the days in which he was absent, an 'L' was entered into the record, but the days were counted as days without rain as the observer was uncertain as to which days the precipitation occurred.

The above mentioned records are difficult to contend with and their use in the analytical computations will lead to futher problems.

Therefore, the records have to be removed from the computational work and this has the effect of reducing the sample size. This situation becomes critical for those stations that already have an abundance of missing records and the resultant characteristic parameters and curves may not be representative of that station.

Another important problem that is difficult to control is the determination of the amount of precipitation lost through evaporation at those stations where the time between observations is not less than 24 hours. This problem is more realistic in southern Alberta as the average temperature during the year is slightly higher than the central or northern regions.

CHAPTER II

STATISTICAL TREATMENT OF PRECIPITATION

2.1 Description of Variables

As outlined in Chapter I, the purpose of this thesis is to develop a series of characteristic curves that would describe the precipitation pattern or behavior for a particular station or geographic area. In order to appreciate the statistical approach to the study, the main variables are introduced at this time and a brief discussion as to their importance is provided.

The main variables used to develop the characteristic parameters are:

1. The number of storms that occurred during the time interval (t_o,t), designated by η_t. The time interval for this particular project is five days.

2. The total amount of precipitation that had fallen during the same time interval of (t_o,t). This variable is represented by the symbol X_t.

The variable η_t will be used to calculate one of the characteristic parameters, λ_1 (the number of storms per some specified time period) while X_t is the main factor in the derivation of another characteristic parameter, λ_2 (1/the amount of precipitation per storm). Further in this chapter, a simplified version of the statistical anal-

ysis developed by previous authors (Verschuren, 1968; Todorovic, 1967) will show how the theoretical distribution function of the random variable η_t, denoted by $F(x/t)$, and the theoretical distribution function of the variable X_t, designated by $F_t(x)$, were derived.

Other variables used in the theoretical development of the stochastic process are listed as follows:

1. The total precipitation during ν storms, X_ν, where $\nu = 1,2..$ ...., denotes the number of storms.

2. The total precipitation during the νth storm, Z_ν, where $\nu = 1,2,......$, denotes the number of storms.

3. A specific amount of precipitation, x.

4. The precipitation intensity at time t, ξ_t, where t denotes the length of the time period.

5. The amount of accumulated precipitation during a single storm up to time t, ζ_t.

2.2 Discussion of the Precipitation Process

Although the science of meteorology has improved in the past several decades, the determination of the outcome of a random variable such as precipitation is likely to be done with uncertainty. While the statistical approach to the prediction of precipitation will not be effective on a short-term basis, it does become useful on long-term considerations. The long-term behavior of precipitation would be of interest to the hydrologist and hydraulic engineer. As the onset of any form of precipitation is dependent on time t with a periodicity of one year, then probabilistic laws are likely to describe the behavior of future precipitation and the statistical tool to illustrate this

behaviour is the stochastic process.

Precipitation is recorded using a depth in inches and is usually in the form of daily, monthly or annual values. Other methods of describing precipitation (especially rainfall) is through the means of hyetographs and mass curves which are obtained from recording rain-gauges, an instrument giving total precipitation during any time interval. The hyetograph, which is a measure of rainfall intensity (ξ_t) against time is illustrated in Figure 1. It will be the initial concept in the statistical description of the important variables. Even with today's progressive techniques and scientific models, rainfall intensity is one variable that is difficult to predict with assurance. Thus future predictions will contain uncertainties. This implies that future hyetographs could not be predicted with certainty, thus ξ_t becomes a random variable for any $t \geq t_o$.

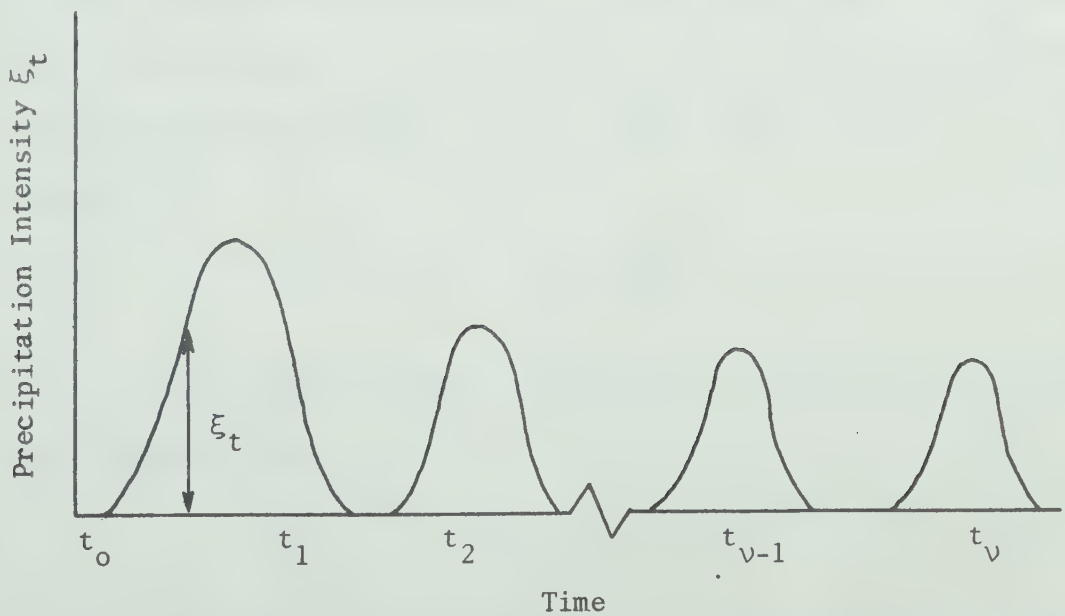

Fig. 1. Hyetograph

Statistically speaking,

$$\{\xi_t;\ t \geq t_o\}$$

is a family of random variables representing a continuous parameter stochastic process for any specific units of time selected from the time interval (t_o, ∞).

While rainfall intensity is very important in different aspects of surface and groundwater hydrology, it is a difficult variable to analyze because of its variation in both time and space. In order to obtain data for intensity measurements, special instrumentation is required (remote-controlled tipping-bucket raingauge) of which there are only a small number in Alberta. Thus it would be difficult to develop a distribution of precipitation that would be representative of the mountain regions or the province as a whole (see Figure 5).

It therefore would be more practical to analyze precipitation in the form of accumulated amounts up to some time t, which from Figure 1 would be the integral of the function ξ_t. Then, X_t, the accumulated precipitation in the interval (t_o, t) can be calculated from the following equation:

$$X_t = \int_{t_o}^{t} \xi_r dr.$$

With X_t being a random variable for every $t > t_o$ and because ξ_t is always positive, then

$$\{X_t;\ t > t_o\}$$

represents a non-decreasing continuous parameter stochastic process.

With a simplified hyetograph as shown in Figure 2 indicating the total precipitation during a stormy period to be concentrated at the end of that period, one can develop a new function from the process $\{\zeta_t;\ t \geq t_o\}$.

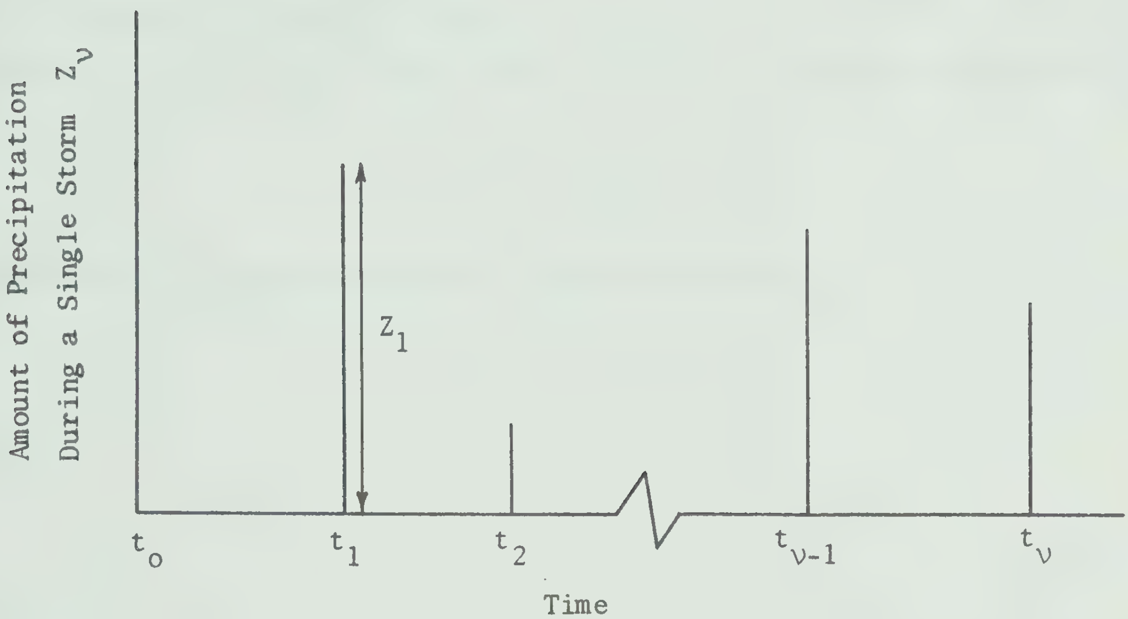

Fig. 2. Simplified Hyetograph

From Figure 2, it is seen that Z_ν is the total precipitation during the νth storm and this can be derived by taking the integral of ζ_t, or

$$Z_\nu = \int_{t_{\nu-1}}^{t_\nu} \zeta_r dr = X_\nu - X_{\nu-1},$$

where X_ν is the total accumulated precipitation until the end of the νth storm. From this process, X_ν becomes a random variable with the expression

$$\{X_\nu;\ \nu = 1,2,3.....\}$$

denoting a discrete parameter stochastic process.

By transposing the simplified hyetograph into a mass curve as indicated by Figure 3, one can establish the number of rainy days or periods in a certain time interval. With reference to the stochastic process $\{X_t;\ t \geq t_o\}$ along with the counting of the number of steps on the mass curve diagram of Figure 3, it can be said that the number of storms (η_t) during the time interval (t_o, t) is a random variable where

$$\{\eta_t;\ t\varepsilon(t_o, t)\}$$

represents a discrete parameter stochastic process.

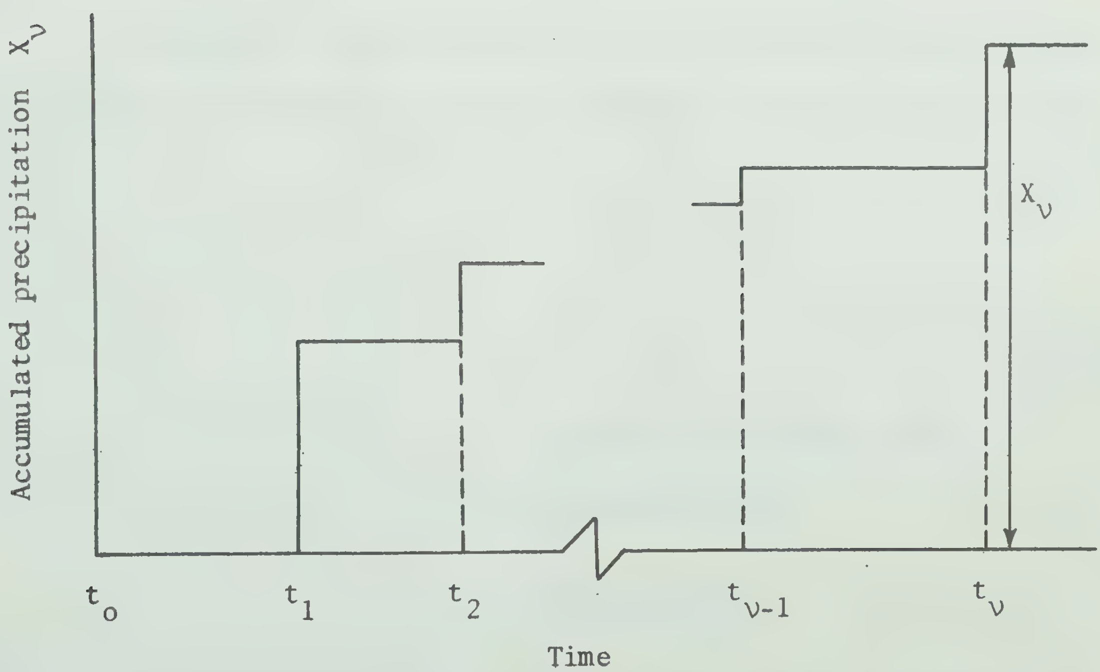

Fig. 3. Simplified Mass Curve of Precipitatiom

In this study, precipitation will be treated as a purely random phenomenon, subject only to periodicity of one year or fractions of a year due to seasonal variations. The concepts "days with measureable precipitation", "days with precipitation", "rainy days, "stormy days" that are used in this study conform with common usage of these expressions.

2.3 Theoretical Development

Extensive studies have been carried out by various authors (Verschuren, 1968; Todorovic, 1967) in regards to the stochastic process $\{X_t;\ t\varepsilon(t_o,t)\}$ and reference should be made to those publications for a more detailed mathematical treatment. It is only intended in this section to show how the theoretical development of the precipitation phenomenon was achieved by these authors so as to provide a guide when observed and theoretical values are compared.

The important concept of the precipitation variable is that of predicting or estimating the average number of stormy days in a time interval (t_o,t). It would be the ability of giving a first approximation or estimation of the number of days with rain or snow to be expected in a time length of a week or a month during some part of the season. Since this concept will be a function of time and other characteristics of η_t, one has to use the theory of probabilities. It is necessary to calculate the following probabilities:

$$P\{\eta_t = \nu\} = p_\nu(t)$$

for every t belonging to (t_o,t) and ν being an integer greater than or

equal to zero in order to estimate the number of storm periods in some interval (t_o,t).

By letting $E_\nu^{t_o,t}$ represent the set of all sample functions of the stochastic process $\{X_t;\ t\varepsilon(t_o,t)\}$, that is, $E_\nu^{t_o,t}$ being the event that exactly ν storms occurred during the interval (t_o,t), then

$$p_\nu(t) = P(E_\nu^{t_o,t}) \ .$$

Considering that these events are exhaustive and mutually disjoint, it follows that:

$$P\{\eta_t = \nu\} = P(E_\nu^{t_o,t}) \ .$$

Before the probability of $E_\nu^{t_o,t}$ can be established, several assumptions and conditions must be satisfied and one condition states that the

$$\lim_{\Delta t \to 0} \frac{\sum_{\nu=2}^{\infty} P(E_\nu^{t,t+\Delta t})}{\Delta t} = 0 \quad \forall \quad t \geq t_o .$$

This implies that the probability of two events occurring in the time interval $(t,t+\Delta t)$ is equal to zero as Δt approaches zero. In general, it states that only one storm at a time can occur and this will be consistent with the usage of daily precipitation or stormy days. For the purpose of the analytical approach used in this thesis, a daily recording of precipitation will be considered as though it occurred from a single storm.

Another condition that affects the probabilities of the events $E_\nu^{t_o,t}$ is given by the expression:

$$\lim_{\Delta t \to 0} \frac{P(E_1^{t,t+\Delta t}/E_{\nu-1}^{t_o,t})}{\Delta t} = \lambda_1(t) \;\forall\; t \geq t_o.$$

In this condition, $P(E_1^{t,t+\Delta t}/E_{\nu-1}^{t_o,t})$ represents the probability that the upper bound of the νth storm will belong to the interval of time $(t,t+\Delta t)$ conditioned on ν-1 storms occurring in the interval (t_o,t). It can be said that the conditional probability depends on t, Δt and ν and can be expressed in the following manner:

$$P(E_1^{t,t+\Delta t}/E_{\nu-1}^{t_o,t}) = \lambda_1(t, \Delta t, \nu-1). \qquad 2.1$$

Based on the assumption that the probability that a storm will occur in the interval $(t,t+\Delta t)$ does not depend on the number of storms up to time t, then the function 2.1 can be reduced to the form:

$$P(E_1^{t,t+\Delta t}) = \lambda_1(t, \Delta t) .$$

The above assumption is not completely valid as there is a certain dependence on the number of storms up to time t, but in order to develop the theoretical equations, independence was assumed (Verschuren, 1968; Todorovic, 1967) and this will be consistent throughout the remainder of this section. With emphasis on this assumption then, in the situation where Δt is very small, λ_1 can be considered to be a linear function with respect to Δt and the expression

$$\lambda_1(t, \Delta t) = \lambda_1(t)\Delta t$$

is valid. Based on the above conditions and assumptions and with the use of generating functions (Verschuren, 1968), it follows that:

$$P(E_{\nu}^{t_o,t}) = p_{\nu}(t) = e^{-\int_{t_o}^{t}\lambda_1(r)dr}\left[\frac{[\int_{t_o}^{t}\lambda_1(r)dr\,]^{\nu}}{\nu!}\right].$$

In other words,

$$P(\eta_t = \nu) = e^{-\int_{t_o}^{t}\lambda_1(r)dr}\left[\frac{[\int_{t_o}^{t}\lambda_1(r)dr\,]^{\nu}}{\nu!}\right].$$

Therefore, if one wished to estimate the average number of storms during the time interval (t_o,t), it would be accomplished by taking the expected value of η_t, or,

$$E(\eta_t) = \sum_{\nu=0}^{\infty}\nu e^{-\int_{t_o}^{t}\lambda_1(r)dr}\left[\frac{[\int_{t_o}^{t}\lambda_1(r)dr\,]^{\nu}}{\nu!}\right]$$

and can be reduced to

$$E(\eta_t) = \int_{t_o}^{t}\lambda_1(r)dr \tag{2.2}$$

It is an established fact that the average number of storms or stormy days during some specified time interval is dependent on the season, thus it can be concluded that $E(\eta_t)$ is a function of time. Before Equation 2.2 can be evaluated, the function $\lambda_1(r)$ must be computed. Unfortunately, it is an unknown function and therefore must be solved from a non-analytical point of view. Let it be accepted that $\lambda_1(r)$ is always positive (since precipitation can never be negative) and that $\lambda_1(r)$ is periodic with respect to seasonal variations. If it is assumed that seasonal variations are small over short time intervals, the function $\lambda_1(r)$ can be approximated by a constant λ_1 over these time intervals. With acceptance of the above argument, Equation 2.2 becomes:

$$E(\eta_t) = \int_{t_o}^{t} \lambda_1 dr$$

and by letting t_o equal zero,

$$E(\eta_t) = \lambda_1 t \quad . \tag{2.3}$$

By considering a number of years of precipitation records at any station or area and with the averaging of the number of storms during a time interval (t_o,t), a value for $\lambda_1 t$ is obtained. The average number of storms per unit of time, represented by λ_1, during the time interval $(0,t)$ is found by dividing $\lambda_1 t$ by that time interval. Hence, the probability that exactly ν storms occur during the time interval $(0,t)$ follows a Poisson distribution and is given by:

$$p_\nu(t) = e^{-\lambda_1 t}\left[\frac{(\lambda_1 t)^\nu}{\nu!}\right] \tag{2.4}$$

with the mean and variance being $\lambda_1 t$ when t_o is considered as zero.

From the definition of the events $E_\nu^{t_o,t}$, it was established that

$$P(E_\nu^{t_o,t}) = P\{\eta_t = \nu\} = p_\nu(t)$$

thus the corresponding distribution function $F(x/t)$ of the random variable η_t has the following form:

$$F(x/t) = P\{\eta_t \leq x\} = \sum_{\nu=0}^{[x]} p_\nu(t)$$

where $[x]$ denotes the greatest integer not greater than x. But from Equation 2.4,

$$p_\nu(t) = e^{-\lambda_1 t}\left[\frac{(\lambda_1 t)}{\nu!}\right]$$

hence,

$$F(x/t) = \sum_{\nu=0}^{[x]} e^{-\lambda_1 t}\left[\frac{(\lambda_1 t)^{\nu}}{\nu!}\right]. \qquad 2.5$$

The individual and cumulative terms of the Poisson distribution can be found in standard tables or programmed through the use of the computer.

With the basic understanding of the theoretical development for the distribution of η_t, one can now establish distributions for other variables since some of the arguments, conditions and assumptions mentioned previously will be applicable in further derivations.

In considering the distribution of x, a specified amount of precipitation, the random variable X_ν, which defines the amount of precipitation during ν storms and is represented by a discrete parameter stochastic process, will be employed in the development of the distribution. Introduce $G_\nu^{t_o,t}$ to represent the set of sample functions of the process $\{X_t;\ t > t_o\}$ that has exactly ν points in the interval (x_o,x). Then the set $G_\nu^{x_o,x}$ will represent the event that the total amount of precipitation will be less than or equal to $x - x_o$ during exactly ν storms and greater than $x - x_o$ during ν+1 storms.

From set theory which states that the intersection of events is a null or empty set and that the union of events is a full set for mutually disjoint and exhaustive events, then for the set $G^{x_o,x}$,

$$G_i^{x_o,x} \bigcap G_j^{x_o,x} = \text{impossible event}$$

and

$$\bigcup_{\nu=0}^{\infty} G_\nu^{x_o,x} = \text{certain event.}$$

From the above definition then,

$$P\{X_\nu \leq x\} = \sum_{j=\nu}^{\infty} P(G_j^{x_o,x}) = C_\nu(x)$$

where $C_\nu(x)$ denotes the distribution function of X_ν for every $\nu = 1,2..$, and the events $G_j^{x_o,x}$ are disjoint and mutually exhaustive.

Before the probabilities of $G_\nu^{x_o,x}$ can be computed, several conditions have to be satisfied with the first suggesting that the

$$\lim_{\Delta x \to 0} \frac{\sum_{\nu=2}^{\infty} P(G_\nu^{x,x+\Delta x})}{\Delta x} = 0$$

implying that the probability that more than one storm will contribute to an infinitely small amount of precipitation is zero.

The second condition expressed by

$$\lim_{\Delta x \to 0} \frac{P(G_1^{x,x+\Delta x}/G_{\nu-1}^{x_o,x})}{\Delta x} = \lambda_2(x) \tag{2.6}$$

has the following explanation; the conditional probability $P(G_1^{x,x+\ x}/G_{\nu-1}^{x_o,x})$ declares that the total amount of precipitation at the end of the νth storm will be between x and $x+\Delta x$ and is conditioned by the total amount of precipitation equal to or less than x during the previous ν-1 storms. It is evident that the conditional probability is dependent on x, Δx and ν and hence can be represented by some function that involves these variables, specifically:

$$\lambda_2 = \lambda_2(x,\Delta x,\nu-1)\ .$$

Let it be assumed that the total amount of precipitation during one storm is equal to or less than Δx does not depend on the total

amount of precipitation during the previous ν-1 storms. Then Equation 2.6 reduces to:

$$P(G_1^{x,x+\Delta x}) = \lambda_2(x,\Delta x) \; .$$

If it is also assumed that λ_2 is a linear function with respect to Δx when Δx is very small, then

$$\lambda_2(x,\Delta x) = \lambda_2(x)\Delta x \; .$$

By accepting the two conditions and the assumptions and through a rigorous mathematical procedure involving generating functions and differentiation (Verschuren, 1968), one finds that,

$$P(G_\nu^{x_o,x}) = e^{-\int_{x_o}^{x}\lambda_2(s)ds}\left[\frac{[\int_{x_o}^{x}\lambda_2(s)ds\;]^\nu}{\nu!}\right]. \qquad 2.7$$

In this case $\lambda_2(s)$ is an unknown, but if it can be assumed that $\lambda_2(s) = \lambda_2$, a constant, by using the non-analytical argument as in the case with $\lambda_1(r)$, then Equation 2.7 reduces to the following:

$$P(G_\nu^{x_o,x}) = e^{-\lambda_2 x}\left[\frac{(\lambda_2 x)^\nu}{\nu!}\right]$$

with x_o taken as zero.

It can be expected that $\lambda_2(s)$ is a function of time as the probability of occurrence of $G_\nu^{x_o,x}$ is different during selected periods or seasons of the year and

$$C_\nu(x) = P\{X_\nu \leq x\} = \sum_{j=\nu}^{\infty} e^{-\lambda_2 x}\left[\frac{(\lambda_2 x)^\nu}{\nu!}\right]$$

or:

$$C_\nu(x) = 1 - \sum_{j=0}^{j=\nu} e^{-\lambda_2 x} \left[\frac{(\lambda_2 x)^j}{j!}\right] . \qquad 2.8$$

Through further mathematical interpretation of Equation 2.8,

$$E(X_\nu) = \nu/\lambda_2$$

and

$$Var(X_\nu) = \nu/\lambda_2^{\,2} .$$

With the distribution of x derived, it is now possible to show how the distribution for X_t, which defines the amount of precipitation during the interval (t_o,t), was developed. For first considerations, choose an event $B_t(x)$ such that,

$$B_t(x) = \{\omega;\ X_t \leq x\} \quad \forall \quad x \geq 0$$

which defines the event $B_t(x)$ as being those values where the amount of precipitation during the interval (t_o,t) is less than or equal to a specified amount of precipitation greater than or equal to zero. By taking the probability of such an event, $P\{B_t(x)\}$, a one-dimensional distribution function of the stochastic process $\{X_t;\ t > t_o\}$ is obtained which will be denoted by $F_t(x)$ where

$$F_t(x) = P\{B_t(x)\} = P\{\omega;\ X_t \leq x\}.$$

On the basis of the set theory of mutually disjoint and exhaustive events, the event $B_t(x)$ can also be written in the following form:

$$B_t(x) = \bigcup_{\nu=0}^{\infty} [E_\nu^{t_o,t} \cap B_t(x)].$$

Thus,

$$F_t(x) = P\{B_t(x)\} = \sum_{\nu=0}^{\infty} P[E_\nu^{t_o,t} \cap B_t(x)]. \tag{2.9}$$

With the aid of Bayes theorem and the logical interpretation of sets and events, Equation 2.9 becomes:

$$F_t(x) = \sum_{\nu=0}^{\infty} \sum_{j=\nu}^{\infty} P(E_\nu^{t_o,t} \cap G_j^{x_o,x}).$$

With the acceptance of the assumptions that λ_1 and λ_2 are constant and that the events $E_\nu^{t_o,t}$ and $G_j^{x_o,x}$ are independent of each other, it can be proven theoretically that the Poisson distribution represents the probabilities of these events (Verschuren, 1968). This allows the distribution function of X_t to be written in the following form:

$$F_t(x) = \sum_{\nu=0}^{\infty} \sum_{j=\nu}^{\infty} e^{-\lambda_1 t} e^{-\lambda_2 x} \left[\frac{(\lambda_1 t)^\nu}{\nu!} \frac{(\lambda_2 x)^j}{j!} \right] \tag{2.10}$$

For x = 0, Equation 2.10 reduces to

$$F_t(0) = e^{-\lambda_1 t}$$

which allows rapid calculations of the probability of zero precipitation during the interval (t_o,t). Computer programs were used to calculate the values for the theoretical distributions shown in this chapter and one program is presented in Appendix F.

To calculate the average amount of precipitation during a time interval of length t, the expected value of X_t is taken, which in effect is the expected value of Equation 2.10. After a lengthy mathematical

procedure (Verschuren, 1968), it was shown that,

$$E(X_t) = \frac{\lambda_1 t}{\lambda_2} \qquad 2.11$$

and the variance was equal to

$$Var(X_t) = \frac{2\lambda_1 t}{\lambda_2^{\,2}} .$$

CHAPTER III

PRECIPITATION MEASUREMENT AND THE TYPE OF RECORDS IN ALBERTA

3.1 Instrumentation

With the scientific advancement in both fields of hydrology and meteorology, there is a need for improved instrumentation as well as the necessity of having complete and continuous information about the elements related to the fields mentioned above. While significant progress (Teweles and Giraytys, 1970) has been made in the development of continuous recording devices in the past decade, most of the devices are confined to urban areas especially at major airports where trained personnel are responsible for their maintenance. Continuous recording devices are also located at experimental and other research stations, but it should be noted that the length of record is usually short, i.e., ten years or less. For all other communities and regions that report precipitation, a standard raingauge is employed and records are maintained by volunteer personnel.

The official Canadian raingauge is a small cylinder with a cross-section area of ten square inches. The gauge is placed in such a position that it is free from all obstructions which might interfere with the catch of rainfall. The gauge is usually located on a level grassy plot, and the rim of the gauge is one foot above the surface of the ground. The rain is caught in the gauge and then measured to 1/100th

of an inch in a glass graduate.

Freshly fallen snow is measured in inches and tenths as it lies on the ground. Observations are made as representative as possible by averaging several measurements and by avoiding snow drifts and wind-swept bare spots. The depth of water resulting from melting newly fallen snow has been taken as one-tenth of the depth or thickness of the snowfall. Thus the total precipitation for any given period is obtained by adding together the total rainfall and one-tenth of the depth of newly fallen snow. Early in the 1960's most principal observing stations were equipped with snow gauges to measure the actual water content, but the data do not extend over a sufficiently long period for inclusion in the total precipitation being reported.

Because of the limited coverage of available raingauge stations, not all precipitation data may be obtained from properly constructed and exposed gauges. Not infrequently the most intense portion of a storm centers over an area where there are no gauges and the greatest rainfall is not measured, or at best is caught in some vessel that may act by chance as a receiver. The value of each such measurement must be judged individually by the person utilizing precipitation data that is not obtained from a raingauge. In some cases he can obtain adequate and satisfactory data whereas in others he may obtain only good guesses. The probable error of such data, of course, will be appreciably larger than that of data of standard gauges, but this reduced accuracy merely diminishes without destroying the usefulness of the results. For the purpose of this thesis, it is accepted that all precipitation data for the stations studied were from the standard raingauge used in Canada.

3.2 Type of Stations

The order of importance of any station recording meteorological parameters is based on the 24-hour period that defines a climatological day, the period of record, and the elements being observed.

First-order stations are maintained and operated by the Atmospheric Environment Service of Canada and are staffed with trained personnel. These stations are located on airports and in larger urban areas and are operated on a 24-hour basis with most of the meteorological parameters or elements being reported. Some of these stations are equipped with recording raingauges, nevertheless, many of the gauges have only been installed in the past decade and records obtained thus far will not be suitable to statistical treatment.

There are 25 first-order stations in Alberta that provide a high quality in precipitation data and the locations are shown in Figure 4. From the map, it can be seen that only a few stations are in the vicinity of the mountains. For those surface-water hydrologists and engineers interested in watershed studies, a lower standard of quality of precipitation data must be obtained from other climatological stations.

At ordinary climatological stations, because the majority of the observers at these stations are co-operative observers, the times for taking the observation varies from station to station. They are encouraged to take the observation twice a day as close to 0800 and 1700 LST as possible. These times may vary as much as one hour on either side of the time indicated. Some stations take the observation only once a day, in the morning, afternoon or evening. Among the ordinary climatolo-

gical stations, the order of importance in regard to the quality of observation is as follows:

1. Stations operated by the Canada Department of Agriculture. At these stations , many of the meteorological elements are recorded by standard gauges as well as by some continuous recording devices.

2. Research stations which are under the joint co-operation of both Federal and Provincial Research Councils. Temperature and precipitation are recorded at these sites using standard gauges only.

3. Experimental Project Stations which are contracted by the Canada Department of Agriculture and which only report several elements using standard gauges.

4. The stations operated by the Alberta Forest Service which are located at the various lookout towers and ranger stations. Except for a few stations, most of these are operated during the summer months only and official records did not originate until 1962. Precipitation is measured in a standard gauge at least twice a day at the tower sites and only once a day at the ranger stations.

5. The stations that located in the various townsites, ranches and etc. with the people in charge being voluntary observers with the minimum of training.

It should be noted that the highest quality of observation of precipitation data would be from experimental watershed studies. However, there are a select few in Alberta and these did not commence operation until the International Hydrological Decade was started in 1965. Figure 5 is a map of Alberta which illustrates the locations of all continuous precipitation recording instruments as well as the year

of commencement or the years of operation.

3.3 Recording of Precipitation Data and Errors Involved

A historical description (Canada Meteorological Branch, 1970a) will now be given outlining the different manners in which the climatological day was defined through the different years of operation.

3.3.1 First-order Stations

In the period from 1878 to May 31, 1924 inclusive there was only one observation a day, and it was taken in the morning at 0700 LST. The climatological day began on day N following the 0700 LST observation, ended on day N+1 at 0700 LST, and was credited to day N+1 for all observed elements.

From June 1, 1924 to December 31, 1932 inclusive if there was only one observation a day and it was taken in the morning at 0700 LST, the climatological day began on day N following the 0700 LST observation and ended on day N+1 at 0700 LST and was credited to day N+1 for all observed elements. If two observations were taken each day, one in the morning at 0700 LST, and one in the evening at 1900 LST, the climatological day began on day N following the 1900 LST observation and ended on day N+1 at 1900 LST and was credited to day N+1.

From January 1, 1933 to December 31, 1940 inclusive if there was only one observation a day and it was taken in the morning at 0630 LST, the climatological day began on day N following the 0630 LST observation and ended on day N+1 at 0630 LST and was credited to day N for most observed elements. If two observations were taken each day, one in the morning at 0630 LST and one in the evening at 1830 LST, the climato-

logical day began on day N following the 0630 LST observation and ended on day N+1 at 0630 LST and was credited to day N for all observed elements.

During the period January 1, 1941 to December 31, 1954 inclusive most stations took four observations daily at the fixed times of 0130, 0730, 1330 and 1930 GMT, with some stations taking one to three observations. The climatological day began on day N following the 0730 GMT observation and ended on day N+1 at 0730 GMT and was credited to day N for all observed elements.

From January 1, 1955 to May 31, 1957 inclusive most stations took four observations at fixed times of 0030, 0630, 1230 and 1830 GMT with some stations taking one to three observations. The climatological day began on day N following the 1230 GMT observation and ended on day N+1 at 1230 GMT and was credited to day N for most meteorological elements. For the next four years until June 30, 1961 inclusive, the above criteria applied for the climatological day, however, the times of observation were taken a half hour earlier, i.e., the new times were 0000, 0600, 1200 and 1800 GMT.

From July 1, 1961 to the present most stations take four observations daily at the fixed times of 0000, 0600, 1200 and 1800 GMT with some stations taking one to three observations. The climatological day begins on day N following the 0600 GMT observation and ends on day N+1 at 0600 GMT with most observed elements credited to day N.

3.3.2 Ordinary Climatological Stations

From 1878 to December 31, 1932 inclusive: (1) if a single observation was taken each morning at 0800 LST, the climatological day began

on day N following the 0800 LST observation and ended on day N+1 at 0800 LST and was credited to day N+1 for all observed elements. (2) if a single observation was taken each afternoon at 1700 LST, the climatological day began on day N following the 1700 LST observation and ended on day N+1 at 1700 LST and was credited to day N+1. (3) if two observations were taken each day, one in the morning at 0800 LST and one in the afternoon at 1700 LST, the climatological day began on day N following the 1700 LST observation and ended on day N+1 at 1700 LST, and was credited to day N+1 for all observed elements.

From January 1, 1933 to the present, the above conditions hold except for several changes. In the case of (1), the climatological day is credited to day N instead of day N+1, and in (3), the climatological day begins on day N following the 0800 LST observation and ends on day N+1 at 0800 LST, and is credited to day N for most observed elements.

Because of the changes in the definition of a climatological day during the time since the first official record, an error will have been introduced, However it will remain fixed and should not cause any concern in the analytical computations. An example of such an error is as follows:

1. At ordinary climatological stations prior to 1932, if it rained after 0800 LST on June 30th, the amount of rainfall was read at 0800 LST on July 1st and the total rainfall was registered for July 1st.

2. Since 1932, if it rained after 0800 LST on June 30th, the guage was read at 0800 LST on July 1st. However, the total amount was credited to June 30th.

Thus, in the case of this thesis where precipitation is studied in five

day intervals, if the years of record overlap the years in which changes were made, then there will be an incorrect count of the number of days with precipitation in some five-day intervals. This will not change the characteristic curves appreciably, but it will have a slight effect on the values for individual months.

As in other types of obervations, those made of precipitation are subject to errors of various kinds and from various sources. There are first the accidental errors in measurement of catch. These errors, however, are small when compared with the daily variations of precipitation and may usually be neglected. A more serious type of error is that due to faulty observations, a type that is sometimes designated as blunders. Faulty observations may involve errors of appreciable size and in storms of relatively large intensity may vitiate important conclusions.

Constant errors are more serious than those mentioned above because they are accumulative. They arise from a variety of sources, such as inaccurate measuring devices, improper exposure, or other continuing causes. Proper exposure and location of the raingauge with respect to adjacent objects are essential for good records of precipitation. If the gauge is improperly exposed the catch may be augmented by water blown off adjacent trees, shrubbery, or nearby buildings, or by splashing from surfaces too close to the receiver. On the other hand, overhanging trees or sheltering walls and buildings may prevent the appropriate quantity of rain from entering the receiver. The gauge should be located away from such objects so that the distance at least equals the height of the object.

The principal biased errors in measuring precipitation are due to three cause: (1) loss by wetting the receiver when transferring precipitation to the measuring cylinder; (2) loss by evaporation; (3) wind effects.

The first error for most precipitation gauges is in the order of .01 inches or less. It may be regarded as an instrumental error and can usually be neglected. Errors due to evaporation depend on a number of factors; geographical and meteorological, location of the station, air temperature, wind, humidity deficit, and instrumental error (design, construction material, etc.). According to Russian investigations (Wiesner, 1970) carried out for a number of precipitation gauge systems, the mean error due to evaporation is 3 to 5 percent of the annual amount for all kinds of precipitation, and .02 inches or less for an individual measurement. This becomes significant in the case of ordinary climatological stations where the times between observation are at least 24 hours.

The maximum errors in measuring precipitation are due to the effect of wind. In the case of rainfall, errors depend on the size of falling drops and the wind velocity. In all rainfall measurements, the effect would have a positive sign, that is, they would lead to a deficiency of rain in the instrument. In the case of solid precipitation (mainly snow), blowing out is observed with an increase in wind speed, and blowing in when the wind speed reaches some critical value and snow drifting takes place. Thus, the errors due to wind are different not only in value but also in sign.

These may be remedied to some extent by the use of shields which

reduce turbulence over the gauges. Without shields, errors of 20 percent have occurred at wind speeds of 10 knots and 50 percent at wind speeds of 40 knots for rain, while for snow, errors of 40 percent and 70 percent have been known at these velocities. However, these are adverse conditions and exposures.

CHAPTER IV

ANALYSIS OF OBSERVED DATA

4.1 Length of Record of Precipitation Stations

Of the thirteen stations selected for analysis in the Province of Alberta, seven were chosen along the southern portion of the province to represent the region from the plains to the mountains, the distance being roughly 200 miles. Four stations spanning the same distance were selected to represent a portion of the central section of Alberta, while two stations in the extreme southern section of the province will be used to study the effects of elevation differences.

The choice of the stations was on the basis that at least 30 years of continuous records were available. It was soon discovered that precipitation records for some stations were missing completely causing the sample size for some months to be as low as 25 years. Nevertheless, it will be shown in Chapter V that reasonable statistical measures can be made with a smaller sample size.

In Tables I and II (Appendix G), the name, location, elevation, period of record and other meteorological information about the elements observed are given for each station (Canada Meteorological Branch, 1967). From Table II, one finds that at a number of stations, snowfall makes a significant contribution to the annual precipitation. As was indicated in earlier chapters, the highest percentage error is

associated with snowfall because of the various factors that affect it at the surface. Thus, it is vital that accurate snowfall measurements be made. More first-order stations would provide higher quality observations, especially in mountainous regions.

4.2 Discussion of Precipitation Data and Associated Problems

In the hydrologist's point of view, all forms of precipitation data are important, especially data from short duration but high intensity storms because of their pronounced effect on hydraulic designs. The variations of precipitation with time and space complicate the matter and demand a diversity of techniques of analysis. One of the problems that arises from analyses of precipitation data is that of sampling. For example:

1. with respect to time, sampling is inferior for those stations that are equipped with standard gauges or non-recording devices.

2. with reference to space, sampling is poor for any area except in high-density gauge networks associated with experimental plots.

Because of the scarcity of continuous precipitation records of any length, daily values were employed in this study. This will introduce systematic errors rather than random errors in the computations where the observed data are used to determine the statistical parameters. These systematic errors are discussed in the next few paragraphs.

Depending on the definition of the climatological day, if a weather system caused rainfall to start at 0800 LST on day N and the

rain continued until 0800 LST of day N+2, then 2 rainy days are counted even though the original weather system should be classed as a single storm. During the summer months when convective storms are most common, a situation arises when several showers or thundershowers pass over a station during separated time intervals of a day and provide a quantity of precipitation. If this occurred during the period of a climatological day, only one reading is taken and it is credited to one rainy day even though two or more storms may have occurred. This raises the problem of classifying or developing a definition for storms as well as establishing a time interval between rainless periods that would decide on the number of storms that actually occurred. This becomes important with respect to some of the random variables used in the mathematical derivation of the theoretical distributions developed in an earlier chapter. Such variables would be the time of the end of each storm and the number of storms during a specified period.

For example, the assumption was made that the probability that a storm will occur in the interval $(t, t+\Delta t)$ does not depend on the number of storms up to time t and that the total amount of precipitation during one storm is equal to or less than Δx is independent of the total amount of precipitation during the previous storms. In this study, a day with measureable precipitation is the smallest time interval used. Thus, if it rained on day 1 and continued into day 2, then the probability of rain on day 2 is not completely independent of the occurrence of rain on day 1. Likewise, the amount of rain on day 2 is not independent of the quantity of rain on day 1. In other words, the assumptions made in Chapter II are not satisfied. However, these

carry-over effects will not extend back for any length of time.

The ability of classifying storms as well as determining the time intervals between storms would be an ideal area for further research as it would involve the meteorological aspects of precipitation processes and storm movements.

By Canadian standards, a day with measureable rain is one on which 1/100th of an inch or more has fallen, and a day with snow is one with at least one tenth of an inch of newly fallen snow. Any amounts less than the above quantities are registered as traces. A trace of precipitation has no appreciable affect on the total amount of precipitation but does have some impact when it comes to counting the number of storms during a specified time period. By climatological standards, a day with a trace is considered as a day without precipitation and this is accepted in the computational work of the thesis. But in actuality, it does mean the under-estimation of the number of days with precipitation.

With the scientific advancement in meteorology, many scientists including hydrologists are taking interest in the meteorological processes that cause precipitation. However, the present recording of data does not give any information about the meteorological systems that caused the precipitation. Precipitation caused by cyclones can be expected to come from a different statistical population than for example, precipitation caused by convection or topography. More generally, high intensity rains, although they may fit the same type of distribution as low intensity rains, probably require different statistical parameters. Yet these different types of precipitation are intermixed

to provide the statistical parameters. In this study, because the precipitation records do not provide this information, no difference is made between the different types of precipitation.

4.3 Discussion of Characteristic Curves.

Thus far, the assumption was made that $\lambda_1(t)$ and $\lambda_2(x)$ were constant in order to develop theoretical distribution functions for some of the random variables involved in this study. The ability to apply the distribution functions as a test of theoretical values versus observed readings will require that the above assumptions be valid throughout the remainder of this project. However, it should be noted that λ_1 [representing $\lambda_1(t)$] and λ_2 [denoting $\lambda_2(x)$] will be subject to seasonal variations in which the periodicity is one year. Since the distribution functions for the random variables do not consider the dependency on time of the parameters λ_1 and λ_2, their values were computed independently at different times of the year in order to obtain the variation of λ_1 and λ_2 with the time of the year, i.e., the season, and the resulting characteristic λ_1 and λ_2 curves.

For any station or area under consideration, the characteristic λ_1 curve can be computed by using any function or distribution that contains only the λ_1 paramter. For example, from Equation 2.4,

$$p_\nu(t) = e^{-\lambda_1 t} \frac{(\lambda_1 t)^\nu}{\nu!}$$

which defines the distribution of η_t, we can obtain the expected value of η_t, denoted by

$$E(\eta_t) = \lambda_1 t$$

and solving for λ_1, we get

$$\lambda_1 = \frac{E(\eta_t)}{t} \, . \qquad 4.1$$

From Equation 4.1, it can be seen that observed data over any arbitrary time interval t can be used to calculate the λ_1 parameter. It is also obvious that as t approaches zero, λ_1 becomes an instantaneous value. With the use of daily precipitation data in this study, t can not be less than one day. While the significance of λ_1 as an instantaneous value decreases with a longer interval of t, it has been observed from the records that the longest number of continual days with precipitation is usually less than five days. Accordingly, the time interval for this study was chosen as five days. Therefore, for all those months with 30 and 31 days, t_o was selected as beginning at 0 hours on the 1st, 6th, 11th, 16th, 21st and 26th while for February, which may have 28 or 29 days, the last interval began with t_o equal to 0 hours on the 21st. It is apparent that the sample size is further reduced, at least by one day for all those months containing 31 days and either 3 or 4 days in February. Since the characteristic curves will represent a series of average values, there will not be any serious deviation on an annual basis. However, for the individual month of February, the λ_1 and λ_2 values will be affected by the absence of the last 3 or 4 days.

Table III (Appendix H) indicates how the λ_1 values were computed from raw data. Once the λ_1 values are calculated they can be plotted against the time of year. The time average value of λ_1 ($\overline{\lambda}_1$) can be superimposed across the points based on Equation 2.2;

$$E(\eta_t) = \int_{t_o}^{t} \lambda_1(r)dr.$$

This equation refers to the mathematical expectation of the random variable η_t, where $E(\eta_t)$ can be considered as the area under the characteristic λ_1 curve for any selected time interval. All the characteristic λ_1 curves for the stations studied are given by Figures 6 to 18.

It can now be seen from the variations in the characteristic curves that the λ_1 function is not constant with respect to time and that the approximate solutions for some of the distributions that were obtained previously by assuming $\lambda_1(t)$ constant during the interval (t_o,t) are less accurate at some times of the year than at others. In some instances, a maximum value of λ_1 occurs in June or early July, a minimum is reached in late July and August and a secondary maximum occurs during September. In other situations, for example Figure 14, the variation in λ_1 is quite large during the period from the end of April to mid-October. Thus, the time interval (t_o,t) during that period must be taken much shorter than for example during March to achieve the same degree of accuracy in determining the frequency of occurrence of some of the random variables. A further discussion of the characteristic λ_1 curves will be given in Chapter V which deals with the comparisons between geographic regions and differences among stations.

The characteristic λ_2 curve is developed in the same fashion as λ_1 except that λ_2 can not be calculated directly but must be derived from the expression,

$$\lambda_2 = \frac{\lambda_1 t}{E(X_t)} \qquad 4.2$$

which assumes that λ_1 and λ_2 are constant over a short interval of time. As λ_1 was calculated using a time period of t equal to 5 days, the same interval must be used to obtain the value for $E(X_t)$. The procedure used to calculate λ_2 is indicated in Table IV (see Appendix H). Computer programs used to do the computational work will be discussed briefly later in this chapter.

The λ_2 values are plotted against the time of year and the graphs for the stations under study are illustrated by Figures 19 to 31. The characteristic λ_2 curve represented by the smooth line will be considered the average value if it is again assumed that changes in the precipitation due to the season may be rapid at any time of the year but that seasonal changes do not occur every day. With the five-day time interval being short with respect to seasonal variations, the λ_2 values can be considered instantaneous values.

An examination of the characteristic λ_2 curves reveals the following properties:

1. The minimum values of λ_2 generally occur during the summer months of June and July.

2. The maximum values are most pronounced during fall and winter.

3. In some cases, λ_2 is nearly constant showing little seasonal effect, whereas in the case of Edmonton (Figure 26), there is a pronounced seasonal distribution.

4. The variation with respect to time for some stations is considerable, i.e., the scatter of the points on some graphs is significant.

Further discussion of the characteristic λ_2 curves will be given in later chapters.

In summary then, the characteristic λ_1 curve can be computed from the average number of precipitation days during five-day periods and that the characteristic λ_2 curve can be derived from the average values of the amount of precipitation during the same time interval, but the value of λ_1 is required for this computation. With restrictions that no data be used outside the five-day time interval, then from the computational methods used in this chapter, λ_1 and λ_2 can be approximately considered as instantaneous values.

4.4 Discussion of Computer Programs

The meteorological data for the thirteen stations were obtained from the Climatological Division of the Atmospheric Environment Service of Canada. The records were compiled on magnetic tape using the card 4 type format (Canada Meteorological Branch, 1970b) in which many meteorological elements were contained in each record. Several computer programs were written to abstract the necessary λ_1 and λ_2 parameters, the first series of programs being developed for the first-order stations and another series for the ordinary climatological stations.

Appendix A gives the program for converting the raw data for first-order stations. At these stations, a trace of precipitation is recorded as a 'T', but for the purposes of this study must be converted to zero. As there is 24-hour coverage of the weather elements at these stations, it is unlikely that any records will be missing unless by clerical omission. The program for calculating the characteristic λ_1

and λ_2 curve parameters for the first-order stations is shown in Appendix B. This program is based on the fact that there are no missing data. Thus, by having knowledge of the initial year of data records as well as the length of such records, all time parameters including years, months and days can be entered as integer unit values. This allows for rapid calculation of the λ_1 and λ_2 values and therefore a minimum use of valuable computing time. A sample of the computer output for such a first-order program is given by Appendix C.

Since it was discovered data records were missing completely and due to the type of observational program at ordinary climatological stations (Chapter I), a special type of computer program was required to correct the raw data. Such a program is shown in Appendix D. In this program, a trace of precipitation, denoted by 'T', is converted to zero while all other alpha characters such as 'L' and 'C' are transcribed to numeric bogus data. Many conversions of data may be necessary resulting in some loss in information.

With the data converted into useable form, a computer program to calculate the characteristic λ_1 and λ_2 parameters for the ordinary climatological station was developed. This program is presented in Appendix E. For those stations that are missing records for whole months and years, bogus data are inserted into the program so as to make the record sequence complete. During the stage of actual computations of λ_1 and λ_2, the bogus data are removed and the values are based on years with true data.

It is apparent then, that ordinary climatological stations with missing records and which employ alpha characters in the observing pro-

gram will, after conversion of data, have smaller sample sizes. This no doubt causes a greater dependence on first-order stations to be more representative of a certain area or region.

The program given by Appendix F provides a solution for the distribution function of the random variable X_t (Equation 2.10). With the values of λ_1 and λ_2 for any station as well as different time intervals (t = 5 days, 10 days, etc.) entered into this program, the output will consist of a table containing the following items:

1. The value of x starting at x = 0 and incremented by 0.05 until the distribution reaches a value of 0.999.
2. The value of the distribution function $F_t(x)$.
3. The increments in $F_t(x)$ for every increment of x.

The values obtained from this program will be used in Chapter VI where a comparison will be made between the observed and theoretical distributions.

CHAPTER V

COMPARISON OF CHARACTERISTIC CURVES

5.1 Introduction

In this chapter, the characteristic λ_1 and λ_2 curves will be analyzed to determine the following effects:

1. The difference in the distribution of precipitation between geographic areas, for example, the difference between Medicine Hat and Coleman or, between the southern and central parts of the province.

2. The discrepancies occurring among first-order and nearby ordinary climatological stations.

3. The significance of topography or elevation difference.

4. The determination of the number of years of record that would give reasonable results.

The comparison of stations will be affected by location, topography, elevation and the type of observing programs and it is expected that these factors will result in the anomalies of some of the characteristic parameters. While it may not be feasible to compare stations of different types in the mountainous regions, it was the intention of the author to provide some insight into the differences of the distribution of precipitation in those regions. The reader should carefully examine any conclusions reached by the author in regards to the effects of the mountains on precipitation. The reader should also bear in mind

the number of stations that are available for study in the mountainous regions and representativeness of any station to the geographic area surrounding it.

5.2 Comparison Between Geographic Areas

5.2.1 Southern Alberta

Consider the group of stations that represent an east-west cross-section of southern Alberta as given by Figure 32. Except for the rolling terrain in the vicinity of Winnifred, there is a gradual change in elevation from Medicine Hat to Fort MacLeod, then an abrupt change due to the foothills followed by a sharp rise into the mountains just west of Lundbreck. In this section, an examination of the characteristic λ_1 and λ_2 curves will be made from Medicine Hat to Coleman to determine the degree of similarity or difference among the entire sequence of stations.

5.2.1.1 Characteristic λ_1 Curves

Generally, the characteristic λ_1 curves for all seven stations along the southern profile have approximately the same seasonal variation, that is, near constant values during the winter months, a maximum value during late May and June, a minimum through July and August and a secondary maximum value in the early part of September. At some stations, there is a secondary minimum value in the month of October.

An inspection of the characteristic λ_1 curves shown in Figures 6 to 12 reveals that the best similarity between stations for the entire year are those of Medicine Hat and Lethbridge. There is some

similarity among the other stations, especially during the summer months. The one station that has a distinct departure from the rest is that of Seven Persons. It should be noted that both Medicine Hat and Lethbridge are first-order stations while the remaining are classed as ordinary climatological stations. It is noticeable that the λ_1 values for the two first-order stations are greater than for the other stations, especially during the winter months, and this can be attributed to the following reasons:

1. Most first-order stations are staffed with trained personnel and a 24-hour coverage of the weather elements is provided.

2. Precipitation readings are taken every 6 hours resulting in 4 observations for every climatological day. This means that more storms, especially the convective type, are likely to be recorded than at ordinary climatological stations where only one observation is taken.

3. The advanced instrumentation at first-order stations will allow the recording of storms that contribute only small amounts of precipitation.

A comparison among first-order and ordinary climatological stations will be made later in this chapter and certain conclusions will be drawn at that time.

To determine the effect of the foothills and the mountains on the distribution of the number of days with measureable precipitation, a comparison was considered between Coleman and two other stations, namely, Lundbreck and Fort MacLeod. Since Lundbreck was located in a valley between the foothills and the first range of mountains, the comparison was made between Coleman and Fort MacLeod as shown in Figure

33. This will enable the reader to determine the contrasts between areas enclosed in the mountains against those areas along the foothills. From Figure 33, it is apparent that there are more days with precipitation at Coleman during the period of October to January while in the period from March to the end of September, Fort MacLeod shows a slightly higher value for λ_1.

Without discussing the broad field of atmospheric circulation and storm movements, it can be presumed that the flow pattern during the winter months provides more days with precipitation within the mountains as shown by the curve for Coleman. In the summer months, the flow pattern provides more days with rain or snow on the east side of the mountains as indicated by the λ_1 curve for Fort MacLeod.

5.2.1.2 Characteristic λ_2 Curves

A study of the characteristic λ_2 curves (representing 1/amount of precipitation per storm) for southern Alberta given by Figures 19 to 25 reveals that for most stations, a maximum occurs during the winter months, a minimum value is reached in the month of June and a secondary minimum happens between late August and September. As was noted in the comparison of the λ_1 curves, a degree of similarity exists among the λ_2 curves for the two first-order stations of Medicine Hat and Lethbridge while some similarity is apparent for the remaining ordinary climatological stations especially during the summer and early fall months. The larger λ_2 values at the first-order stations can be accounted for by the fact that more days with measureable snow or rain greater than a trace is possible with continuous observations. The

explanations given for the departures in the characteristic λ_1 curves among the different stations applies as well to the λ_2 curves.

It is interesting to note that for some ordinary climatological stations, for instance, Coleman, Lundbreck and Seven Persons (Figures 20, 24 and 25, respectively), the λ_2 curve is almost constant throughout the entire year. This implies that the same amount of precipitation, whether it be rain or the rainfall equivalent of snow, can be expected during any one stormy day or generally, any period of time. In mountainous country, especially at higher elevations, snowfall can contribute to the total annual precipitation as much as rainfall, and with reference to Table II, it can be seen that the amount of snowfall at Coleman and Lundbreck when converted to rainfall equivalent is slightly below the annual rainfall. However, in the case of Seven Persons, the annual snowfall is just slightly more than one-half of the annual rainfall. This suggests that the λ_2 curve for this station is not representative of the conditions in this area. Also from Table II, the average annual snowfall for Medicine Hat (48.7 inches) is nearly the same as that of Seven Persons (47.6). However, when comparing the λ_2 curves given by Figures 19 and 20, there is a significant difference between the two stations. On the basis of the differences in the λ curves as well as the data given in Table II, a comparison between first-order and ordinary climatological stations was necessary and will be given later in this chapter.

Having discussed the effects of the mountains on the number of days with measureable precipitation by comparing the λ_1 curves for Coleman and Fort MacLeod, it can now be determined what changes might

occur in the amount of precipitation per storm by analyzing the λ_2 curves for the same two stations. With reference to Figure 34, the λ_2 curve for Coleman is nearly constant with an average value of 4.0 whereas in the case of Fort MacLeod, the λ_2 values vary from 8.2 in January to 2.8 in June. From this graph, it can be assumed that the amount of precipitation per storm inside the mountains of southern Alberta (using Coleman as a representative station) is almost constant from one month to the next showing some independence of the flow pattern. On the lee side of the mountains, using the λ_2 curve for Fort MacLeod, one finds that the precipitation per storm during the winter months is constant with a value of approximately 6.5. However, towards the latter part of May and most of June, the λ_2 value is at a minimum suggesting that the maximum amount of precipitation per storm occurs at this time. It is often the case where storms moving through southern Alberta or in the vicinity cause upslope or orographic conditions to extend from the plains region to the first range of mountains. As a result, extensive precipitation can be expected with the heaviest amounts in the foothill areas west of Lethbridge as indicated by the lower values of λ_2 at Fort MacLeod and Lundbreck. With a constant λ_2 value for Coleman during the summer months, it is apparent that the orographic precipitation in this part of the country has its greatest variation on the lee side of the mountains.

5.2.2 Central Alberta

A group of stations as shown by Figure 35 were chosen to represent a portion of central Alberta where the distance from the mountains

was similar to the stations of southern Alberta. It also was important that these stations have at least thirty years of record. It is intended in this section to compare the characteristic curves of the central Alberta stations and then determine the contrasts between different zones of Alberta which will be considered in succeeding sections.

An examination of Figure 35 reveals that the graph is similar to Figure 32 inasmuch that the slope of the land from Edmonton to about 20 miles west of Edson is gentle, then rises abruptly to form the foothills and is very steep on the west side of Entrance. It may be noted that the first range of mountains in this part of the province is at a higher elevation than in the southern sections and that Jasper is entrenched in a deeper north-south valley as compared to Coleman's east-west valley. As in the discussion for southern Alberta, the characteristic λ_1 and λ_2 curves will be examined from Edmonton to Jasper in order to develop some contrasts as to similarity or reveal any significant departures.

5.2.2.1 Characteristic λ_1 Curves

The behavior of the characteristic λ_1 curves for the four stations along the central zone indicates a distinct variation for the different seasons of the year. Most pronounced is the maximum value that occurs during mid-June, July and in some instances, the early part of August. Minimum values appear during the spring months and for Edmonton only, in October. At Edson and Entrance, a rather constant value prevails during the winter months.

A comparison of the characteristic λ_1 curves illustrated by Fig-

ures 13 to 16 indicate that the best similarity exists between Edson and Entrance although Edson has a slightly higher value of λ_1 during June and July. With reference to Figure 35, one can see that the range of the foothills lies between Edson and Entrance. During the months of June and July, many convective storms originate on these foothills and are usually fully grown by the time they reach Edson thus producing more precipitation at this station than possibly at Entrance. This would account for some or if not most of the differences between the λ_1 values for June and July for the two stations mentioned above.

The degree of similarity between Edson and Entrance is of particular interest as Edson is a first-order station while Entrance is classed as an ordinary climatological station although Edson did not become a first-order station until 1960. Considering the distinct departures that occurred between the different types of stations for southern Alberta, it can only be concluded that:

1. The area around Entrance has the same slope, exposure and orientation to weather storms as Edson.

2. The volunteer observer at Entrance was conscientious and reported all storms producing precipitation.

3. The characteristic λ_1 curve for Entrance is not representative of the area and should be either higher or lower depending on the manner in which weather storms affect this area.

4. The period of record used for Edson was prior to the station becoming a first-order station. It therefore is reasonable to accept that any similarity would comply with the fact that both stations were ordinary climatological stations before the year 1960.

To determine which of the above points is most prominant would require further research and therefore will not be pursued in the course of this thesis.

A continued examination of Figures 13 to 16 reveals that both Edmonton and Jasper (first-order stations) show a considerable variation throughout the entire year. However, there is a good resemblance among all four stations during the months of May to September inclusive. This implies that on the average, most of the central region, at least west of Edmonton, has the same number of days with measureable precipitation during the summer months. During the winter months, say November to March inclusive, it appears that more days with snow or rain occur inside the mountains and across the plains region as compared to the foothills area.

To get a better understanding of the effects of the mountains on the distribution of the number of days with measureable precipitation, the λ_1 curves for Jasper and Entrance were compared against each other as demonstrated by Figure 36. From the graph, it can be seen that from the period of mid-March to early September the two stations have about the same number of days with precipitation. This suggests that the mountains have little effect on the weather storms passing through this part of the province. During the fall and winter period (mid-September to mid-March), there is a marked difference in the λ_1 values between Jasper and Entrance. This indicates that the mountains have a significant effect during this period resulting in more days with measureable rain or snow at Jasper. As mentioned earlier in this chapter, without becoming involved with the mechanics of meteorological circulation and

storms, it can be presumed that the dominant flow pattern over this particular area during the fall and winter months provides more days of meteorological activity within the mountain range as compared to the lee side. This is demonstrated by the constant low values of λ_1 at Entrance during the fall and winter season.

5.2.2.2 Characteristic λ_2 Curves

Similar to the graphs displaying the characteristic λ_1 curves, there is a noticeable seasonal variation in the λ_2 curves for those stations across central Alberta. Generally speaking, from Figures 26 to 29, there is a maximum λ_2 value during February and March, a minimum value extending across June and July and a secondary maximum during October. It can be deduced from these graphs that the bulk of the annual precipitation occurs in the summer months in the form of rain while lesser amounts are contributed by snowfall.

Because of the topography of the area between Edson and Entrance, it was precluded that there would be differences in the amount of precipitation per storm. However, the λ_2 curves for the two stations (Figures 27 and 28) were almost in total agreement with each other which is significant since there was a high correlation between the λ_1 curves. This in effect suggests that both the λ_1 and λ_2 curves for the two stations could be combined and the average curves obtained might be representative of an area extending from the lee side of the mountains to some location between Edson and Edmonton.

A further inspection of Figures 26 to 29 reveals that for the period of May to September inclusive, there is a good correlation be-

tween Edmonton, Edson and Entrance. This implies that any meteorological activity covering the area between the three stations is apt to produce the same amount of precipitation for any time interval in the summer season. In the winter months, except for Edson and Entrance, there is a significant difference among the stations. The λ_2 values for Jasper from April to September are slightly higher than for the remaining stations and this indicates that smaller amounts of precipitation are produced per storm within the mountainous areas. To obtain a better concept of the effects of the mountains on the λ_2 values, a comparison was made between Jasper and Entrance as illustrated by Figure 37.

Referring to the elevation chart of Figure 35, one finds that the elevations of Jasper and Entrance are similar. However, a sharp and high mountain range separates the two stations. From a meteorologist's point of view, this would suggest that those storms approaching from a westerly direction would leave most of its precipitation within the mountains, whereas those storms causing an easterly upslope flow would have most of their precipitation left on the lee side of the mountains. An inspection of Figure 37 shows that except for January, Entrance receives more precipitation per storm for the entire year. While the λ_1 values for Jasper from mid-September to mid-March were much higher (Figure 36) than at Entrance for the same period, that is, there were more days with measureable precipitation in the mountains, the λ_2 values for Jasper were also larger. The smaller amounts of precipitation at Jasper suggest the assumption that the mountains in this area have little effect on the storms travelling from a westerly direc-

tion or that Jasper lies in a so-called rainshelter where most of the precipitation is deposited either upstream or downstream of the station location. Extensive studies of the meteorological circulation and storm movements would have to be undertaken to discover the impact of the mountain range on the λ_2 curve in this particular area.

5.2.3 Contrasts Between Southern and Central Zones

There will be situations where hydrologists, engineers and other environmental scientists will be concerned about the distribution of rainfall over a large area or across the entire length of a major river. If important diversion of water were necessary to augment supplies to a populous region, then the rainfall distribution between specific areas may become the prime subject of research. If such situations did arise, it would be essential to compare various stations of a specific zone and eventually develop a statistical mean value of all stations. This mean value of the characteristic parameters would approximate the conditions of that particular zone. With knowledge of the similarities or differences between areas, the engineers in water management could justify the need to transport water, if necessary, and determine the extent and the amount of water that would have to be transported.

In this section, the stations of southern and central Alberta will be combined into a general set of characteristic curves and a discussion will be made between the different sets of curves. The reader should be cautioned that the combination of first-order and ordinary climatological stations may result in unrealistic values and the validity of the characteristic curves for the stations used will be left to

his judgement. Also, the combination of stations in the mountainous areas with those stations on the plains region will result in misleading information. Nevertheless, it is only intended to show whether contrasts do exist between geographic zones and to what extent the information could be used.

5.2.3.1 Characteristic λ_1 Curves

Considering the southern Alberta zone initially, the statistical average and standard deviation of the characteristic λ_1 values of all seven stations combined is given by Figure 38 and it is immediately noted that significant variations occur from May to November. The graph suggests that more days of measureable precipitation take place during the summer months indicating that the bulk of the annual precipitation for this particular zone is in the form of rainfall. An interesting note about Figure 38 is that during the winter months of December to April, the standard deviation is fairly large. This may be due to several reasons:

1. The large variations in the λ_1 values among the first-order and ordinary climatological stations will cause an overall lower average λ_1 curve for all stations combined as well as a significant deviation.

2. Some snowfall is sublimated in the foothills area because of the chinook winds that prevail during the winter months. As a result, those storms producing small amounts of snow may not be recorded by the stations that report precipitation only once a day.

3. Partly related to point 2, the lack of proper instrumentation for the recording of small snowfall amounts at ordinary climatolo-

gical stations will cause measurement differences among stations in the same geographic area.

4. Combination of the stations in the vicinity of the mountains with those of the plains will cause some deviation as the mountain stations receive more days with snow because of their higher elevations.

An examination of Figure 39, which is the combined average and first deviation of the characteristic λ_1 values for the four stations in central Alberta, reveals that generally there is a marked seasonal variation for the entire year. Minimum values are reached in the months of April and October while the maximum values extend across most of June, July and August. This summer maximum may be an indication of the high frequency of storms (whether orographic or convective) that happen in this portion of the province. Another feature that is noticeable is the small deviation of the curve for all stations during the period from May to September inclusive. This suggests the following items:

1. On the average, most orographic storms are large enough in areal extent to cover a large portion of the central zone.

2. The number of convective showers or thundershowers are evenly distributed among these central Alberta stations.

In the winter months of October to March inclusive, there is a large deviation from the average curve. Since the combination of stations involved both first-order and ordinary climatological stations, it would appear that the deviation in the number of days of measureable precipitation (generally snow) during that winter period is mainly due to the type of observing program, topography and storm path movements.

In order to study the contrast of the number of days with rain

or snow for both southern and central Alberta, one must refer to Figure 40 which displays the characteristic curves for the regions concerned. It is plain to see that the combined stations of the central zone receive more days with measureable precipitation than the southern part of Alberta except for April where the λ_1 values are similar. The contrast of any significance between the two zones occurs during June and July. In that interval of time, the southern region indicates a decreasing value of λ_1 and the central region has its maximum values. This contrast may be partially explained by the higher frequency of convective showers that take place in the central areas through the summer months. Both regions experience a minimum number of days with precipitation in October and it appears that the winter storms affect the northern half of the province more often than the southern half if the λ_1 curves are any indication.

5.2.3.2 Characteristic λ_2 Curves

With some knowledge of the variations of the characteristic λ_1 curves and the reasons thereof for the two regions under study in this section, one can now examine the characteristic λ_2 curves for the same regions and thus evaluate the quantity of precipitation per storm.

Considering the southern half first, one notes from Figure 41 that there is some seasonal variation of the λ_2 curve, but it is not as pronounced as the λ_1 curve. Although a minimum value of approximately 4.0 exists during June and September, the curve shows constant values for the period of May to September inclusive. During the cooler months of October to April, much of the precipitation is in the form of snow

and higher λ_2 values indicate smaller amounts of precipitation (rainfall equivalent of snow) per storm. The large deviations during those cooler months can be explained by the same reasons given for the deviations in the λ_1 curve where the measurement of snowfall, the averaging of stations from the mountain and plains areas, and the different types of observing programs are considered.

A study of Figure 42 shows that the seasonal variation of the characteristic λ_2 curve for the central stations is similar to the curve given in Figure 41. The constant values from June to August suggest that, on the average, most of the area from Edmonton to Jasper receives the same amount of precipitation per storm whether it be of the convective or orographic type. It is interesting to note from Figure 39 that the λ_1 curve for central Alberta is at its maximum during the same period and with a little smoothing, a constant λ_1 value can be obtained. This suggests that the λ_1 and λ_2 curves for central Alberta, shown by Figures 39 and 42 respectively, would be representative of an extensive area west of Edmonton for the three summer months. Another feature of Figure 42 is that in the month of February, the least amount of precipitation can be expected over the central region. The significant deviation over the winter months is due to the lesser amounts of precipitation received at Edmonton and Jasper as compared to Entrance and Edson where topopraphy has a greater effect.

It is interesting to note from Figure 40, which gives the comparison of the λ_1 curves for southern and central Alberta, that the differences between the two curves (except for April and May) were quite large. But, when the λ_2 curves for the two zones (Figure 43) are com-

pared, the variation between the two curves is not as large. Using the values from Table II, the combined average annual precipitation for the southern stations is 16.58 inches and is 18.86 inches for the central stations. The variations in the λ_1 and λ_2 values for the two areas would account for the differences in the annual average precipitation. A specific study of Figures 40 and 43 shows that the λ_1 values for April are similar, but the λ_2 curves indicate a difference of 2.5 for the same month. Also, in the period of mid-July to mid-August, the λ_2 values are identical but a large departure exists between the λ_1 curves for the same period.

Even though the two areas of central and southern Alberta have, on the average, the same number of days with measureable precipitation during the month of April, the southern region receives more precipitation per day. This could be due to the greater amounts of wet snowfall in the mountains and more rainfall across the plains area resulting from slightly warmer temperatures in April over southern Alberta.

The other interesting anomaly is the situation in late July and early August where the greatest difference occurs between the λ_1 curves yet the λ_2 values for that period are similar. During this time of the year, a larger number of the storms are convective in nature and it appears from Figures 40 and 43 that while the central area of the province receives approximately twice as many storms, the amount of precipitation per storm is nearly the same. It can only be presumed that the fewer storms across the southern regions provide a greater abundance of precipitation or that the type of observing program has a serious effect on the characteristic curves of the stations in these regions.

Some insight to the above mentioned anomalies may be revealed when a comparison is made between a first-order and nearby ordinary climatological station later in this chapter.

After discussing the characteristic curves for the different zones in Alberta using the average values of a series of stations in each zone, it might be of interest to examine the curves for individual stations of each zone, especially near mountain areas where the majority of the rivers in Alberta have their origin. Referring to the elevation charts in Figures 32 and 35, one notices that Coleman and Jasper are within the first mountain range (looking from the east) and that the stations of Fort MacLeod and Entrance lie on the east side of this first mountain range.

Considering the lee side of the mountain first, one can see by Figure 44 that the characteristic λ_1 curves for Fort Macleod and Entrance show a difference during the period from early June to mid-September and are reasonably similar for the remainder of the year. In the early summer months, major storms affect the Province of Alberta and usually last for several days providing a significant amount of precipitation. From Figure 44, it is demonstrated that Entrance has more days with precipitation for the month of June. This suggests that those storms affecting Alberta during this time have a tendency to last a little longer in the central sections of the foothills.

In July and August, convective storms are more common and the larger λ_1 values at Entrance suggest that a higher frequency of these storms occur in the central parts of the foothills area as compared to southern sections. The frequency of the convective storms will be

dependent on the amount of vegetative growth in the area, evaporation and transpiration and the weather systems moving through the province during those two summer months. For the remainder of the year, it appears that both stations record nearly the same number of days with measureable precipitation.

When making note of the amount of precipitation per storm for both Entrance and Fort MacLeod, it is surprising to discover from Figure 45 that the λ_2 curves are nearly identical. This gives rise to several suggestions:

1. The similarity is purely coincidental.

2. The two stations receive the same amount of precipitation per storm.

3. Because of the departures in the λ_1 curves, the differences of exposure, orientation and topography between the two stations may have a significant effect on the λ_2 curve, making them similar.

Discounting point 1 as being by chance and accepting points 2 and 3, one can say that generally on a long term basis, the lee side of the mountains receive the same amount of precipitation per storm whether it be southern or central Alberta. Point 3 would require further research to determine the effects of exposure, orientation and topography on the characteristic λ_2 curve.

With some knowledge of the distribution of precipitation on the lee side of the mountains, a study can also be made about the distribution within the first range of mountains. It should be remembered that the effects of topography on the characteristic curves of mountain sta-

tions will be important and any comparison made may be unrealistic. However, from a practical point of view, the comparison of mountain stations may provide some information about the distribution of precipitation in this difficult area of study. The townsites of Coleman and Jasper were selected and the characteristic λ_1 and λ_2 curves for both stations are given by Figures 46 and 47, respectively. Coleman is an ordinary climatological station while Jasper is classed as a first-order station.

An examination of Figure 46 indicates that except for the period of mid-March to the end of May, Jasper has considerably larger λ_1 values than Coleman, but by Figure 47, it also has greater λ_2 values. This suggests that while it has more days with precipitation than Coleman, the amount of precipitation per storm is less. It is obvious from Figures 46 and 47 that Coleman receives more precipitation during the period of March to May inclusive and this is confirmed by Table V which gives the 30-year monthly precipitation for those months mentioned above.

TABLE V

30-YEAR MEAN MONTHLY PRECIPITATION (INCHES) FOR COLEMAN AND JASPER

Station	Month		
	March	April	May
Coleman	1.26	1.56	2.02
Jasper	1.10	0.99	0.63

Several interesting deductions of the distribution of precipitation inside the mountains can be made with reference to the geographic

zones of Jasper and Coleman, the characteristic curves of Figures 46 and 47, and Table V. Some are as follows:

1. Higher λ_1 values at Jasper indicate the passage of more storms through that region.

2. Large values of λ_2 for Jasper suggest small amounts of precipitation from the storms indicating that the townsite is in a protected area (meteorologically speaking).

3. Fewer storms moving through the southern mountain areas provide more precipitation as compared to the central regions.

4. Coleman, being at a higher elevation will likely receive more precipitation and may not be representative of the southern mountains in general.

5. The effects caused by the different types of observing programs in relation to measurement of precipitation.

From the factors listed, it is obvious that more stations should have been selected within the mountainous areas and greater emphasis placed on topography, location, orientation, exposure and meteorological circulation of the atmosphere. Limitations were imposed on the number of stations chosen for analysis and an involved meteorological aspect was not the scope of this study.

5.3 Study of First-order and Ordinary Climatological Stations

In the previous sections, considerable mention was made in regards to the types of observing programs at different stations. This was due to the variations occurring in the characteristic curves among stations within reasonable proximity of each other. These variations

or differences were most inherent among stations along the southern section of Alberta. In some instances, the discrepancies were explained through meteorology, in other cases, topography and elevation were assumed to be factors, but in some situations it was presumed that errors in measurement or the type of observing was the key factor.

With the definitions of the different types of stations given in Chapter IV, a comparative study can now be made between a first-order and a nearby ordinary climatological station. Consider the two localities of Medicine Hat and Seven Persons, separated by a distance of 15 miles and having elevations of 2365 and 2480 feet above mean sea level, respectively. As indicated by Figure 32, the profile between the two localities appears smooth, thus topography should not be a major factor. It could be assumed that the distribution of precipitation should be the same for both stations.

A comparison of the stations shown in Figure 48 indicate that a large variation exists between the two λ_1 curves. While both stations show the same seasonal variation, the λ_1 curve for Medicine Hat has a factor range of 1.2 to 2.0 greater than the λ_1 curve for Seven Persons for the entire year. Due to the nature of the type of precipitation over Alberta, especially in the case of convective storms, one could expect some differences between nearby stations, say in the order of 10 to 15 percent. With a variation as high as 50 percent, one can only assume that the nature of observing is the over-all factor in accounting for such differences between Medicine Hat and Seven Persons. First-order stations are staffed with trained personnel providing continuous coverage of the meteorological elements while most ordinary clima-

tological stations are maintained by voluntary observers who are not compelled to provide continuous observation. It therefore can be assumed that many storms providing small amounts of precipitation are not recorded at ordinary climatological stations because of wind-action, evaporation and other effects. This could be a partial explanation for the discrepancies between the nearby stations.

An inspection of Figure 49 reveals that the λ_2 curve for Seven Persons has the same range of difference that was discovered in the comparison of the λ_1 curves. Referring to Table II, the mean annual precipitation for Medicine Hat and Seven Persons is 14.29 inches and 13.79 inches respectively, but the total number of days with measureable precipitation varies from 93 days at Medicine Hat to 66 days at Seven Persons. Rather than accept that more precipitation is expected per storm at Seven Persons because of the lower λ_2 values, the author is compelled to believe that the observing programs at some ordinary climatological stations fail to report the total precipitation at their station

Therefore, for the characteristic curves of an ordinary climatological station to be representative of the precipitation in its surrounding area, and adjustment factor may be required. The magnitude of such a factor and where it can be applied would require extensive studies with an intricate precipitation network. But for practical purposes, such curves could be used for rough estimates.

In the event that the comparative study of the characteristic curves for Medicine Hat and Seven Persons was an extreme situation and to avoid using Seven Persons as an example for all ordinary climatolo-

gical stations, an inspection was made of the λ_1 and λ_2 curves for Fort MacLeod and Lethbridge. The elevation chart of Figure 32 shows that there is a minor difference in elevation between the two stations and the topography is of a slight rolling nature. While there will be some effect of topography on the distribution of precipitation at Fort MacLeod because of its nearness to the foothills, one can assume that the distribution of precipitation at the two localities should have a high degree of similarity.

Referring to Table II again, the mean annual precipitation for Lethbridge is 17.23 inches and for Fort MacLeod is 17.67 inches while the number of days with precipitation for the former is 110 days and only 88 days for the latter station. This difference in the number of stormy days is in agreement with Figure 50 which illustrates that the λ_1 values for Fort MacLeod are less than in the case of Lethbridge except for the latter part of August and all of September. The greatest differences between the curves are during the winter months and it can be presumed that the high frequency of chinook winds in this area may have a considerable impact on snow measurement. The difference in values in the period of late July to early August can be attributed to the number of convective storms that are recorded, especially those storms producing small amounts of rain but are lost due to evaporation if times between observations are not less than 24 hours. It can also be seen from Figure 50 that the same seasonal variation occurs at both stations.

Since Lethbridge and Fort MacLeod have approximately the same mean annual precipitation, one would expect that with Fort MacLeod

having the lower λ_1 values, it would have lower λ_2 values than Lethbridge and this is generally confirmed by Figure 51 which gives the comparison of the λ_2 curves for both stations. Similar to the λ_1 curves of Figure 50, the greatest difference of the λ_2 values occur during the winter months when a greater part of the precipitation is snowfall.

As can be seen from both λ_1 and λ_2 curves for Lethbridge and Fort MacLeod, making adjustments to the characteristic curves for an ordinary climatological station would be difficult without further study because of the time and space variability of precipitation. An important factor that has arisen from the study of the curves for both Lethbridge and Fort MacLeod is that the measurement of snowfall may not be precise at ordinary climatological stations in or near mountainous regions. As a result, the amount of snowfall may be under-estimated due to the problem of measurement and this may have a serious effect on the conclusions drawn by the hydrologist or hydraulic engineer who makes full use of precipitation amounts in his calculations of river runoff.

From the analysis of both sets of stations, it has been shown that the measurement of precipitation at ordinary climatological stations is inferior to that of first-order stations and a certain degree of caution should be exercised when employing precipitation records for important analytical considerations in hydrology.

5.4 Effects of Elevation

Besides its relationships with time and area, precipitation displays variations with other parameters. These may be physiographic, like height, slope, aspect, and distance from the sea, or dependent on airmass properties. On occasions, rainfall has been related to one

particular variable but with limited success. As in most situations, there is more than one significant variable. For example, an empirical relationship can be established between rainfall and elevation, but its use is limited to a specific locality and cannot be generalized without involving some other physiographic variable. It has been shown by authors in meteorology (Wiesner, 1970) that there is an increase of precipitation with elevation. The main feature about rainfall, as a mountain range is ascended, is not the higher intensities received but the longer periods of rainfall, which are often of light intensity.

The need to study the variation of precipitation with elevation in Alberta stems from the fact that most important watersheds are embedded in the mountainous regions. An important future concept would be to find the degree of variation of precipitation with height and the critical elevation where the maximum amount of rain or snow is likely to occur. Then this information could be related to the elevations of any watershed under study. Eventually, a representative distribution of precipitation could be obtained without involving an extensive and expensive precipitation gauge network.

In mountainous regions, the variations in precipitation make assessment particularly difficult especially because sites for rain-gauges are few because of inaccessibility and are often non-representative. For this particular study, the problem of evaluating the effects of elevation on precipitation for the Alberta mountains was quite difficult as the author wished to compare two stations within the mountains that would have the following criteria:

1. The stations be separated by a distance not greater than 10

or 15 miles.

2. The elevation difference being 1000 feet or greater.

3. That at least 25 to 30 years of continuous precipitation records were available.

4. The time period of the records be the same for both localities.

5. That both stations were of the first-order type providing continuous observations.

To find a series of localities in Alberta that would satisfy all the above criteria was impossible. The two stations that could satisfy at least three points of the criteria were those of Cardston and Carway. Some information about the stations is given in Tables I and II. The elevation contour map of the Cardston-Carway region illustrated by Figure 52 shows that Cardston is exposed in a clockwise direction from northwest to southeast and that Carway is embedded in a set of hills with elevations between 4400 and 4500 feet above mean sea level. Ideally, for a better precipitation-elevation study, the station of Carway should have been located on the ridge with elevations of 4750 feet and greater. The elevation chart of Figure 53 is a north-northeast by south-southwest line between the two townsites and it indicates that Carway is slightly protected by the ridge to those winds from the north and northeasterly directions. In the winter months, those wind directions have a tendency to cause upslope orographic precipitation, especially snowfall. Thus more snow is expected to accumulate on that ridge as compared to Carway.

In order to determine the effect of elevation on precipitation

between the chosen stations, the characteristic λ_1 and λ_2 curves were produced and compared as given by Figures 54 and 55, respectively. It can be seen from Figure 54 that both stations experience the same seasonal variation and, except for the time interval of February to May inclusive, the λ_1 values are nearly identical. The lower values of λ_1 for Carway for that time interval may be due to some meteorological phenomenon, such as prevailing flow of moist or dry air. By examining Figure 55 and making reference to Figure 54 and Table II, the following deductions can be made:

1. During the period of June to September inclusive, both stations receive the same amount of precipitation.

2. For the month of May only, Cardston is expected to receive slightly more precipitation as it has a higher λ_1 value but the same λ_2 value as compared to Carway.

3. For the remaining months of October to April, the differences in the λ_2 values in relation to the differences in the λ_1 values suggest that Carway receives more precipitation per storm which during that time interval would be mainly in the form of snow.

The above information is confirmed by the mean annual values given in Table II which notes that at the townsite of Carway, the mean annual values of total precipitation and snowfall are higher, but that the mean annual total number of days with measureable precipitation is less than in the case of Cardston.

While the elevation difference between the two localities is only 685 feet, the change in mean annual precipitation amounted to slightly more than two inches indicating that greater amounts of pre-

cipitation can be expected with height or near mountain ranges. While this concept has been proven by many scientists in meteorology, the purpose of this section is to indicate how the time distribution of precipitation is affected by changes in elevation. This was accomplished through the development of the characteristic curves which demonstrated, at least for this particular area, when the differences in precipitation amounts could be expected.

It should be remembered that the concepts developed in this section may not be applied in general because of the space variations of precipitation and also that the two stations studied were of the ordinary climatological type. Because of the nature of observing at these stations, randon and systematic errors are possible and this may alter the situation to some degree. Nevertheless, those hydrologists wishing to develop a complete precipitation distribution over a mountainous watershed must consider the elevation factor and may have to be satisfied with any type of station.

5.5 Number of Years of Record Required

Realizing that this study would involve the statistical treatment of daily precipitation for a number of stations, it was felt that at least 30 years of records would be required as a larger sample size would provide a smoother representation of the variable in question. For those stations that contain records of considerable length, a reasonably smooth set of characteristic parameters could be obtained. But there are many stations (including Alberta) where the length of record is short and may not be continuous during some periods. For those sta-

tions, some suitable length of record would have to be determined which could cause a variable to be representative or characteristic of that station.

To provide some insight into the question of the length of record required to obtain a suitable graph of λ_1 or λ_2 versus time of the year, the record for Edmonton was divided into two sections; one of ten years from 1959 to 1968 and one of fifteen years from 1954 to 1968. The values of λ_1 were computed for both periods and compared against the 31-year time average while the λ_2 values were computed for the fifteen-year period and also checked against the longer period of record. The comparisons are plotted on Figures 56, 57 and 58.

An examination of Figures 56 and 57 shows a close similarity between the ten-year and fifteen-year time average curves with the 31-year average curve. However, the fifteen-year curve has a closer and smoother fit to the longer period curve than that of the ten-year record length. Referring to Figure 13, it is apparent that the shorter periods of record show a greater scatter about the time average curve. Several important features are evident with the inspection of Figures 13, 56 and 57 and some are as follows:

1. The similarity between the λ_1 graphs for the periods 1938-1968, 1954-1968 and 1959-1968 indicate that no significant change in the number of measureable days with precipitation has occurred since 1938 and it may be safe to assume that little change has occurred since observations began in 1880.

2. There is no doubt that a very long period of record would eventually provide a reasonably smooth curve. By inspecting Figure 57

and noting the close similarity between the curves, it is quite practical to say that the averaging of data for a shorter period of record will provide a curve that is a close approximation of the smooth curve that would have been obtained if an infinite period of record were available.

3. With the concept of using shorter periods of record for the development of characteristic curves for different variables associated with the precipitation phenomenon, one can now perform analysis on newer stations that may be located in remote areas and therefore allowing a greater coverage for any specific zone or region.

In Figure 58, the characteristic λ_2 curves for Edmonton are shown as the smoothed curve of the computed points for the period 1954-1968 and the time average curve that was drawn in Figure 26. Both curves have the same seasonal distribution and can be classed as being similar. For the period 1954-1968, the λ_2 values except for the months of July to October inclusive are higher indicating a smaller amount of precipitation per storm for the most recent period. Since the precipitation gauge for Edmonton has not been moved any considerable distance in the past fifteen years and since there is a very good agreement of the λ_1 curves from Figure 57, it can be assumed that no major change in the climate has occurred. However, the possibility of a small fluctuation or oscillation in the climate should not be overlooked.

It is unlikely that the difference in the amount of precipitation is purely a matter of chance and some other factors must be the cause for this change. It would be difficult to speculate as to what these factors are, but one could take into account the rapid expansion that

the City of Edmonton has experienced in the past fifteen years or the increased number of pollutants that exist in the airmass as of recent times. As was noted with the comparison of the λ_1 curves for Edmonton for the different time periods, there is a greater scatter about the mean value curve of the λ_2 values for shorter periods of record. In order to establish the effect of the differences in λ_2 between the periods of 1954-1968 and 1938-1968 on the expected amount of precipitation [$E(X_t)$] for Edmonton for any time interval, it was decided to divide the records into two parts and compare the resulting curves against those of the longest period.

Consequently, the records were divided into the time periods of 1938-1953 and 1954-1968 and compared with the total length of record consisting of the years 1938-1968. The characteristic λ_1 and λ_2 curves were drawn on Figures 57 and 58. However, the plotted points apply only to the interval 1954-1968. It can be seen from Figures 57 and 58 that while all curves generally have the same seasonal variation, there are differences and similarities during different portions of the year for all three time periods. Most noticeable is the departure in the λ_2 values of Figure 58 during the first six months of the year and then a good similarity in the values from July to October and finally some small differences in November and December. It would appear that the smaller amounts of precipitation per storm during 1954-1968 as compared to the previous sixteen years of record for the first half of the year would suggest that possibly a small oscillation in the climate had occurred over Edmonton in the past 30 years. A verification of such an oscillation would require further study of other meteorological param-

eters.

To determine the degree of error that would result in the expected amount of precipitation for Edmonton for any time interval using a shorter length of record, the months of June and September were selected and the data presented in Table VI.

TABLE VI

EXPECTED AMOUNT OF PRECIPITATION FOR EDMONTON FOR JUNE AND SEPTEMBER

	Period		
Month	1	2	3
	1938-1953	1954-1968	1938-1968
June	3.03	2.47	2.87**
September	0.96	1.23	1.13

** Amounts are expressed in inches.

The amounts shown in Table VI were obtained from the λ_1 and λ_2 values extracted from the curves for the three time periods. The percentage error of periods 1 and 2 with period 3 for the month of June is 5.6 and 13.7 percent, respectively. For the month of September. the error of periods 1 and 2 with period 3 is 15.1 and 9.5 percent, respectively. The different values demonstrate that significant deviations occur in the expected amount of precipitation when shorter periods of record are used. If necessary, certain adjustment factors may have to be developed and applied to the analyzed data of records of shorter length.

5.6 Summary

With the development of the characteristic λ_1 and λ_2 curves for

the precipitation variable for any station, some knowledge can be attained as to how the precipitation is distributed in time. By comparing stations within a certain zone or region, the λ_1 and λ_2 curves can give some information about the distribution of precipitation in space. Grouping of stations that contain some similarity and show the same seasonal variations allows the classification of geographic zones and therefore a valuable concept in the time and space distribution of precipitation over an extensive area. Further study by the hydrologist of the precipitation phenomenon with other meteorological parameters will enable him to determine the overall effect of the distribution of precipitation on the course of any river from its watershed to the eventual termination in any large body of water. The characteristic curves can also be useful in determining, to some degree, the type of precipitation that occurs for any specific locality.

Any project study involving the accurate determination of runoff using precipitation data will require that these data be reasonably accurate. Such accuracy can only be attained if the station reporting precipitation has quality control, that is, the station is staffed by trained personnel who provide a continuous observation using the most recent and sophisticated instrumentation. Of the several types of stations in Alberta, the most common are the first-order and ordinary climatological stations. Comparative studies of the above stations when in close proximity of each other reveals that some departures do exist and in some cases were quite excessive. This is mainly due to the differences in observational techniques, therefore, some caution should be heeded when data are being used from ordinary climatological

stations. It has been shown that some of these stations are likely to report an undercatch of the true precipitation. Whenever possible, some correction factor should be applied to ordinary climatological stations to make their records compatible with first-order stations.

Knowledge of precipitation in mountainous terrain, especially over major watersheds, is vital in the determination of excess runoff for flood forecasting and other purposes. It is generally accepted that precipitation amounts per storm increase with height to a critical elevation and then decrease above that level. The study of the effects of elevation on precipitation becomes important in those remote areas where the hydrologist has only a few stations available, but by accounting for the changes in elevation, a reasonable distribution of precipitation can be obtained.

The determination of the minimum number of years of record required to give reasonably smooth λ_1 and λ_2 curves was undertaken as many stations in Alberta have short lengths of continuous records. It has been shown that a record length of fifteen years is satisfactory in providing characteristic curves that would be representative of a particular station. Although appreciable deviations did occur when a shorter period of record was used as compared to a longer length of data, an adjustment factor may be introduced or an improved method of smoothing could be initiated. It should be remembered that all of the smoothing of the curves in this project was through the use of moving means and then a final correction by eyesight.

And finally, it should be noted that while some meteorological theory was used to explain anomalies between stations, especially in

mountainous regions, these same anomalies may have been the result of the type of observing program carried out at the station in question. It is impossible at this time to say what factor contributed most to the anomalies, but it is hoped that further research into the meteorological aspects of precipitation and as well the amount of undercatch of precipitation at ordinary climatological stations will provide more concrete evidence as to the behavior of precipitation distributions in Alberta.

CHAPTER VI

COMPARISON BETWEEN OBSERVED AND THEORETICAL DISTRIBUTIONS

6.1 Introduction

Further to the development of the characteristic λ_1 and λ_2 curves based on the techniques illustrated in Chapter IV and with the extended discussion of the curves in Chapter V, it will be the purpose of this chapter to determine the theoretical distributions for certain stations by the substitution of λ_1 and λ_2 values in the theoretical equations derived in Chapter II. The theoretically derived distributions will be compared to the actual distributions and then it can be determined whether the λ_1 and λ_2 values extracted from the average curves will provide distributions that are in close proximity to the population distribution compared to the case where the actual computed values of λ_1 and λ_2 were used. In some instances, especially over short intervals of time, the scatter about the average curve may be appreciable. Thus one should expect noticeable discrepancies between observed and theoretical distributions. In the tables referred to in this chapter, for simplicity, abbreviations will be used; 'obsvd' representing the observed data and 'theor' denoting the theoretical distributions based on λ_1 and λ_2 values derived from the characteristic curves.

In the succeeding sections, comparisons will be made for single stations, groups of stations, for differences between nearby stations

and it will be investigated whether or not the theoretical distributions can be used to determine the precipitation regime over geographic areas. The ultimate goal of such a study is that of providing a first estimate of precipitation behaviour over a region that has little or no precipitation and hydrological instrumentation. This may be accomplished through a certain method of transposition of characteristic curves from a nearby station and accommodated with the proper adjustment factors which might include elevation, topography, orientation, hemispheric location and some knowledge of the climate.

The tables providing the comparisons of the observed and theoretical distributions and which are referred to in this chapter are given in Appendix I.

6.2 Number of Storms During a Specified Time Period

From the examination of the graphs illustrated by Figures 6 to 18, it was discovered that the function $\lambda_1(t)$ computed from the distribution of $E(\eta_t)$ was not constant with respect to time. With reference to the general equation for the mathematical expectation of the random variable η_t (Equation 2.2),

$$E(\eta_t) = \int_{t_o}^{t} \lambda_1(r)dr$$

it appears that, since $E(\eta_t)$ is merely the area under the characteristic λ_1 curve over the time interval considered, then the time average value $(\bar{\lambda}_1)$ of λ_1 over the time interval can be employed. In other words, the characteristic curves represent the time average values of λ_1.

Therefore,

$$\int_{t_o}^{t} \lambda_1(r)dr = \bar{\lambda}_1(t - t_o)$$

and which allows the use of the Poisson distribution with parameter $\bar{\lambda}_1 t$ (if t_o is taken as zero) for the distribution of η_t.

To evaluate the effectiveness of the Poisson distribution in describing the precipitation phenomena, the observed frequencies and the theoretical frequencies were computed for the first five days of each month for Edmonton as given by Table VII and for the first five days of January for the four stations along the central Alberta zone as shown by Table VIII. From Figures 6 to 18, it is noted that a seasonal variation persists at every station and in most cases a sudden change occurs in the λ_1 values between May and July. In order to test the applicability of the theoretical equation during the time interval of a sudden change in λ_1, observed and theoretical frequencies were computed for a 20-day period. For Edmonton and Edson, this period was from June 1 to 20, while May 21 to June 10 was selected for the stations of Entrance and Jasper. The results are shown in Tables IX and X, respectively. In all cases, the λ_1 values (shown in the tables) are average values over the time interval from the characteristic λ_1 curves of Figures 13, 14, 15 and 16.

An overall inspection of Table VII reveals that there is a reasonable agreement between the observed and theoretical distributions. The areas of disagreement are those for the frequency of occurrence for one day where the theoretical equation often gives an over-estimation of occurrence and in the case for two days, the theoretical distribution shows an under-estimation of the number of occurrences. It is presumed

that, by choosing the actual λ_1 value computed from the distribution of $E(\eta_t)$ instead of the λ_1 value selected from the characteristic curve, there will not be much change in the differences between the observed and theoretical distributions.

The reader can recall from previous chapters that when daily precipitation data was used, there is the inevitable over-estimation of the number of storms when weather systems persist for more than one day. It is often the case in the analytical treatment of precipitation data that when two or more continuous days with measureable precipitation were counted, it was suspected that one major storm was occurring at the time. This type of situation is very prevalent over western Canada where well-developed weather systems move slowly eastward during certain seasons of the year and provide several days of precipitation to any specific locality. This condition has some bearing on the assumption made in Chapter II which stated that the probability a storm will occur in the interval $t+\Delta t$ does not depend on the number of storms up to time t. This assumption of independence was required so that the theoretical equations could be developed (Verschuren, 1968; Todorovic, 1967). With any weather system lasting for several days, it can be accepted that the probability of rain in the interval $t+\Delta t$ is dependent on whether or not it rained up to time t. In other words then, the Poisson distribution equations developed in Chapter II based on the assumptions must be considered as an approximation only if daily precipitation records are used. As a result, differences in the observed and theoretical distributions should be expected.

Any improvement of the theoretical distribution equations to

provide results similar to the actual or observed distributions will require the incorporation of meteorological theory involving precipitation regimes. Referring to Table VII again and accepting that the departures between observed and theoretical frequencies are small, then one can consider at least for the station of Edmonton that the theoretical distribution is a rather good approximation of the number of days with precipitation during the first five-day period in each month. It is certain that the theoretical equations would provide reasonable results for the remainder of the year.

To determine whether the theoretical distribution equations are a good approximation for the other stations, the λ_1 values for the interval January 1-5 were extracted from the characteristic curves for Edson, Entrance and Jasper, and applied to the proper equations. The month of January was chosen because the λ_1 value for all four stations displayed a constant value as compared to the other months. The results are given in Table VIII. It is noted that for the higher λ_1 values, for instance Edmonton and Jasper, there is a good agreement between observed and theoretical frequencies; but for the lower values of λ_1 for Edson and Entrance, there is a poor agreement, especially for days of zero precipitation and for one-day occurrences. The discrepancies can be accounted for by the fact that since precipitation gauges often to fail to record small amounts of precipitation, it must be expected that, when observed and theoretical distributions are compared, the observed distribution will generally show a larger number of days with no precipitation.

Mentioned previously was the problem of a varying λ_1 during

marked changes in the seasonal variations and what effect it would have on the theoretical distribution. Table IX and X give the results for the 20-day interval that included the maximum change in λ_1 for the four central Alberta stations. The λ_1 value used is the average value of the characteristic curve of the selected time intervals and should be a good approximation of the actual λ_1 value based on the distribution of $E(\eta_t)$, therefore a reasonable agreement should be expected between the observed and theoretical frequencies. Inspection of Tables IX and X indicates that while differences in frequencies do occur, the departures are not serious and it can be accepted that the use of an average value of λ_1 will be representative for those situations where there is a marked seasonal variation. It is possible that a longer period of record would show a better correspondence between observed and theoretical distributions for a λ_1 curve that has marked seasonal variations.

Another interesting test is that of evaluating the effectiveness of the theoretical distribution for longer intervals of time. Thus Tables XI and XII were developed to compare the observed and theoretical distributions for the central Alberta stations during the 30-day period of January 1-30. The λ_1 values shown in the tables were extracted from the characteristic curves and should be similar to the averaged λ_1 values obtained from the observed data. This suggests that discrepancies between observed and theoretical frequencies should be minimized.

In the situations for Edmonton and Jasper, both having high λ_1 values, there is a reasonable agreement between the observed and theoretical frequencies. In the case of Edson and Entrance where lower

λ_1 values were experienced, there is disagreement between the two distributions. The differences in the latter case might be attributed to the following factors:

1. The Poisson distribution equations are ineffective at low λ_1 values.

2. The observing program at Edson (a first-order station) under-estimated the number of days with measureable precipitation, especially snowfall during January. Actually, Edson did not become a first-order station with full instrumentation until February of 1960 and therefore a good possibility exists that during the period of record 1929-1959, many stormy days with small precipitation amounts could have been neglected.

3. With Entrance being an ordinary climatological station and since it has been shown that such stations are likely to under-estimate the number of stormy days, then a constant error over the years of record may have some significant effect on the observed frequency.

4. The larger departures between the observed and theoretical distributions are for zero and one-day occurrences of precipitation. The theoretical equations based on their assumptions are unable to account for the anomalies.

Discounting point 1 as unlikely and considering that points 2 and 3 have some effect, the emphasis should be placed on point 4 to account for the differences in the two distributions. There is a good likelihood that the probability of rain or no rain on any day has its greatest impact on those days with zero precipitation and on one day occurrences.

In summary then, it should be accepted that the Poisson equation provides a theoretical distribution that is a good approximation to the precipitation regime for an area with the condition that the characteristic curves developed for the area are representative of the precipitation variable.

6.3 Amount of Precipitation for a Given Number of Storms

The analytical development of λ_1 and λ_2 values, the graphical representation of the characteristic curves, and the introduction of theoretical distributions to approximate precipitation behaviour in the previous portions of this study has now led to the most important concept of hydrological and water resource planning. It is the determination of the probability of occurrence of a quantity of precipitation less than or equal to a fixed amount in a selected time interval during a particular time of the year. The important application of the theory presented in previous chapters will be in the situations where some knowledge will be required about the amount of precipitation to be expected in an area with no instrumentation or any precipitation records. This could be accomplished through the transposition of characteristic curves of a nearby area and these curves would provide the parameters necessary to determine the probability of an expected amount of precipitation.

The theoretical distributions of the random variable X_t for the four central Alberta stations were computed with the aid of a computer program and the resultant distributions were compared to the observed values. The month of January was selected as the λ_1 and λ_2 values for

all the stations were reasonably constant in comparison to the remaining months. Thus t_o was chosen as January 1st., 0 hours. For this particular month, the average values of λ_1 and λ_2 selected from the characteristic curves are as follows:

	λ_1	λ_2
Edmonton	.385	12.25
Edson	.212	6.35
Entrance	.213	8.00
Jasper	.350	8.50

The results of the calculations and the comparisons are given in Tables XIII to XVIII where X_t was considered for the time intervals $t = 5$, 10, 15, 20, 25 and 30 days. Basically, there is a fair agreement between the observed and theoretical frequencies for all the time intervals selected. A detailed study of the Tables XIII to XVIII reveals the following properties:

1. For Edmonton, the agreement between the observed and theoretical frequencies is good for all time intervals which demonstrates the importance of quality observations from a first-order station that has proper instrumentation.

2. For the remaining three stations, there is reasonable agreement during the time intervals of $t = 5$ and 10 days. For the longer time intervals of t ($t = 15$, 20 days, etc.), differences between observed and theoretical frequencies became more noticeable. Suggestions for the departures in the cases of Edson and Entrance were given in the previous section. An explanation for the departures for the

station of Jasper is difficult, but it may be speculated that the λ_2 value chosen for January is not representative for the entire month. Comparison of the frequencies for the remaining months for Jasper would be required to determine the overall representation of the λ_2 curve for this station and the mountainous area in the vicinity.

One more measure of testing the expected value of precipitation over a specified location is that of plotting the theoretical and observed values of $E(X_t)$ and $Var(X_t)$. For Figure 59, the stations of Medicine Hat and Edson were selected for the month of January. It can be seen that the average values of X_t are in close correspondence with the theoretical values and there is also a good agreement in the variances for the first 15-20 days of the month. However, the variance of the observed values become much greater as the time interval increases suggesting a greater dispersion about the mean value than the theory would indicate.

6.4 Comparison Between Nearby Stations

In chapter V, the comparison of first-order and ordinary climatological stations was initiated because of the noticeable differences in the characteristic curves between Medicine Hat and Seven Persons. The two stations are separated by a distance of 15 miles and are of different types. To further examine the precipitation distribution of this particular area in Alberta and observe the behaviour of the theoretical distributions, the month of August was selected for the two stations as it was the only month which did not have any missing records for either station. The comparisons of frequencies are given

in Tables XIX to XXIV.

The prevalent features of the tables are as noted:

1. For all time intervals, the observed frequencies of days with zero precipitation for Seven Persons are greater than Medicine Hat.

2. With the exception of t = 5 and 10 days, generally there is a poor to fair agreement between observed and theoretical frequencies for both stations. Some of the disagreement, especially for longer time intervals, may be due to the fact that average values were being used for a varying λ_1 value.

3. The theoretical distribution under-estimates the days of no precipitation especially for the station of Seven Persons. Considering that Seven Persons is an ordinary climatological station and subject to certain types of error because of its observing program, it would almost be feasible to use the theoretical distribution of Medicine Hat as being more representative of the conditions at this locality.

4. In the case of Medicine Hat, as t increases, the theoretical distribution has a tendency to show a higher frequency for greater amounts of precipitation than does the observed distribution.

The above arguments apply to the month of August only and should not be construed to be representative of these stations for the remaining months of the year. Time and space limitations of this thesis did not allow a complete inspection of a station for the entire year.

6.5 Applicability of the Theoretical Distribution to Geographic Areas

The concept of combining stations where some lie within the mountains, others on the lee side and the remaining stretched towards the

plains region may be questionable. However, considering that the mean annual precipitation (from Table I) for the central Alberta stations are similar to each other and the fact that the characteristic curves were also reasonably similar, it would seem practical to have such a combination of values to be representative of any large area.

The construction of Tables XXV to XXX were by the following procedures:

1. The observed average is the average of the frequencies of all four stations used in the central Alberta section.

2. The theoretical average denoted with index 1 was evaluated through the use of λ_1 and λ_2 values extracted from Figures 39 and 42, respectively.

3. The theoretical average denoted with index 2 was taken from Tables XIII to XVIII in which the theoretical frequencies were averaged among the four stations.

Although there is a slight under-estimation by the theoretical distribution for days with zero precipitation, one can accept that a reasonable correspondence exists between the two distributions. Also, with a high correlation between the two theoretical averages, it is apparent that the combined characteristic curves are representative for the area between Edmonton and the mountains. Therefore, it can be suggested that if characteristic curves in a region indicate a similar type of precipitation distributions, then such curves may be combined thereby giving a single representative curve which will be applicable to the entire region and possibly to a greater extent.

6.6 Test of the λ_1 and λ_2 Values with Published Meteorological Data

In earlier sections, the λ_1 and λ_2 parameters as obtained from the characteristic curves were used to derive the theoretical distributions. To determine the representativeness of the characteristic curves compared to published data, the Edmonton and Seven Persons records were chosen. Only monthly average values of λ_1 and λ_2 could be used to compare with published meteorological data.

The results are shown in Figures 60 to 63. For Edmonton, the comparison of the average monthly values of the number of days with measureable precipitation using the λ_1 parameter shows close agreement with the monthly averages obtained from published data (Canada Meteorological Branch, 1967). The average monthly values of the amount of precipitation using the λ_2 values shows reasonable agreement with the monthly averages obtained from published data. The discrepancies for the months of June, July and December may be due to the smoothing techniques used to develop the characteristic curves and which may have some effect on the values of λ_1 and λ_2 for those months. At Seven Persons, there are some differences in the number of days with precipitation but the average amount of precipitation shows a good similarity.

The meteorological published averages are usually based on a 25- to 30-year record length within the period from 1930 to 1960. In some instances, the averages are based on record lengths as low as ten years. In the case of Seven Persons, the monthly averages of some precipitation parameters were based on a period of record of less than ten years. In this thesis, all characteristic curves were derived from record lengths of 25 to 31 years within the period 1924-1968. The differences in the

period of record and the number of years within the period explains some of the discrepancies between published monthly values and values obtained from the λ_1 and λ_2 curves.

CHAPTER VII

APPLICATIONS

7.1 Introduction

For the hydrologist and hydraulic engineer who may have considered meteorology as simply "it rained" or "it did not rain", the discussion of the study of such concepts as the number of storms during a specific time interval, the amount of precipitation per storm, characteristic curves, observational programs, theoretical distributions and others has probably prompted him to consider where such concepts would be of practical value. Having observed that the use of the λ_1 and λ_2 parameters derived from representative curves provided a reasonable correspondence between observed and theoretical distributions, the specialist can then incorporate the theoretical functions (with certain probability limits) into the analytical treatment of the precipitation variable. Most often the engineer will be interested in precipitation amounts received at the ground. Thus the distribution of the random variable X_t (the amount of precipitation during a specified time interval) would be of first priority, therefore discussion will be mainly confined to the application of this randon variable.

7.2 Irrigation

In this area of study, several types of professionals would be

concerned about the expected amount of precipitation at any specified area. For instance, the agronomist's interest would involve the amounts of water required for different types of crops because of the seasonal effects of evapotranspiration whereas the engineer and hydrologist would have to incorporate the seasonal fluctuations of precipitation into the overall design of an irrigation system. To demonstrate how the distribution function of the random variable X_t can be used as a guide, consider a case of row-crop farming in the Medicine Hat region where the known minimum amount of water required to raise a crop will aid in determining the risk of crop failure without irrigation.

Let it be assumed that the minimum amount of water required to raise corn in the first 20 days of August is equal to 2.00 inches. If the characteristic curves for Medicine Hat are representative for the area surrounding the city, then the λ_1 and λ_2 values are 0.246 and 5.40, respectively. With these values and $t = 20$ days substituted into Equation 2.10, the probability that $x = 2.00$ inches or less is .95. That is to say, 95 percent of the time, irrigation water must be supplied in order to maintain a crop of corn that requires a minimum of 2.00 inches during the first 20 days of August.

For the situation described above, let it be presumed that 1.60 inches can be supplied by irrigation and that the remaining 0.40 inches must be available from precipitation during those 20 days. The distribution function for X_t shows that crop failure would occur 20 percent of the time if 0.40 inches is substituted for x in Equation 2.10. Therefore, from the above example, irrigation planning can be based on a certain loss-to-risk basis that would provide a cost-benefit ratio

suitable to all parties that are affected by an irrigation system.

7.3 Precipitation - Runoff Relationship

The importance of precipitation - runoff relationships is of concern to that engineer or hydrologist who is in charge of providing an accurate flood forecast especially in those regions where there are no hydraulic control structures and where river discharge data are inadequate to determine peak flows in streams and rivers. River forecasts are also required for estimating inflow to reservoirs in order to permit the most efficient operation for flood control and other purposes. In addition, there is an increasing demand for day-to-day forecasts of river stages and discharges by those interested in navigation, water supply, stream pollution, and many other related fields.

The tools of an engineer involved with forecasting include precipitation - runoff relations, unit hydrographs, routing methods, recession curves and stage - discharge relations. Because of the importance of the time factor, great stress must be placed on the development of forecast procedures that will enable flood warnings to be issued at the earliest possible time. Any forecasting service is dependent on adequate data. The development of the river - forecasting procedures requires historical data, while the preparation of operational forecasts requires sufficient current information.

Most often a design storm, commonly referred to as the probable maximum storm, is placed over a drainage basin and after losses due to infiltration, seepage and other causes are accounted for, the excess rainfall is converted to river discharge at the concentration point.

In other words, the probable maximum flood can be obtained from the probable maximum precipitation by subtracting loss rates and adding the snow melt with due regard to relationships with time.

Judgements must be made in interpreting data and in reaching estimates of the probable maximum precipitation as currently there are no set of rules, graphs, and procedures whereby one can proceed step by step and necessarily derive an acceptable estimate. The application of the following methods may provide a reasonable possible estimate and they are:

1. The storm model approach.

2. The maximization and transposition of actual storms.

3. The use of generalized data or of maximized depth, duration, area data from storms. These are derived from convective or orographic storms.

4. The use of empirical formulae determined from maximum depth, duration, area data, or from theory.

5. The use of empirical relationships between the variables in particular valleys.

6. The statistical anlyses of extreme rainfalls.

Unless the hydrologist or engineer is highly specialized in meteorology and has a capacity to apply some of the methods mentioned above, reliance must be placed on other statistical procedures that afford a shorter time in analytical treatment. It has been the purpose of this project to suggest that a reasonable degree of certainty can be obtained through the application of the characteristic λ_1 and λ_2 curves in combination with the distribution function for X_t when the statisti-

cal comparison of different design storms is necessary.

The engineer who feels that depth, duration and area are the major factors in which a design storm would be used in determining a maximum probable flood, then the probability that the precipitation will be equal to or less than some amount x can be obtained through the use of Equation 2.10. The value of the depth of precipitation (x) and the duration (t) are the substitution parameters selected from any area-depth-duration curve. For example, if t is chosen as 5 days and the curve indicates a value for x of 0.50 inches, then the probability of this amount of precipitation being exceeded is equal to $1 - F_t(x)$, or 0.15 if the λ_1 and λ_2 values are 0.246 and 5.40, respectively. An important note to be remembered is that, since the values vary because of seasonal variations, the probability of occurrence of the selected design storm will not always be the same during different times of the year.

7.4 Weather Modification

The importance of the knowledge of precipitation distributions in the area of weather modification should never be neglected because of the future implications resulting from the modification of weather systems. Before any modification program is deemed successful or not, many facts must be searched by the scientists who are attempting to induce more precipitation from the cloud structure of a storm system.

It can readily be seen that if the characteristic curves are accepted to be representative for an area or region for a considerable length of time, then the two variables:

1. the number of storms during a specified time period, and

2. the amount of precipitation per storm,

are going to assist the scientists in determining both short and long range effects of any precipitation modification program. The characteristic curves will have the capacity of directing both the time and spatial requirements of precipitation modification, once it has been proven to be effective and feasible.

CHAPTER VIII

CONCLUSIONS AND RECOMMENDATIONS

8.1 Conclusions

The sequence of this thesis, commencing with the development of the theoretical equations through the modification of the Poisson equation, the derivation and discussion of characteristic parameters, and then the application of the theoretical distributions to describe the precipitation phenomenon, has provided the following conclusions:

1. The assumptions used to derive the theoretical equations were not completely valid, but this problem is not serious and it is reasonable to expect the theoretical distributions to be a good first approximation of the behaviour of precipitation.

2. The two most important variables required to make the theoretical distributions applicable are:

   a. The number of storms during a specified time interval.

   b. The amount of precipitation per storm.

The two variables are subject to seasonal fluctuations with a periodicity of one year and are represented by two characteristic curves that can be used to obtain an estimate of the probability of occurrence of an amount of precipitation in any time period.

3. The attempt to describe the anomalies between characteristic curves for different stations with meteorology should not be accepted

CHAPTER VIII

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3. The attempt to describe the anomalies between characteristic curves for different stations with meteorology should not be accepted

as conclusive as these anomalies can also be the result of the type of observing program at each station. There was no attempt to determine which factor had the greater influence.

4. Although longer periods of record lend themselves to better statistical treatment, a minimum of 15 years of record in conjunction with proper smoothing techniques can be used to develop characteristic curves that would be representative of a longer period of record.

5. The discrepancies resulting from the comparison of the observed and theoretical distributions when daily precipitation data are used is greater for ordinary climatological stations than in the case of first-order stations.

6. The theoretical distribution, in most cases, under-estimated the number of days with zero precipitation and also over-estimated the larger precipitation amounts for a given number of storms.

8.2 Recommendations for Further Research

Due to the time limitations and the unavailability of more data, several problem areas of precipitation behaviour and regime could not be explored. Some of the more intersting ones are as follows:

1. To improve the assumptions used in the derivation of the theoretical equations where daily precipitation is employed, the dependence of rain in the interval $t+\Delta t$ on whether or not it rained up to time t should be considered. Also, a smaller time interval of precipitation measurement could be used. The use of hourly or 6-hourly records would alleviate the problem in the classification of "storm" and "stormy days". Such a procedure would require the need of continuous

recording devices as well as a properly chosen time interval that would define a storm or the break between storms.

2. In some applications of hydraulic engineering practices, there is a requirement for the knowledge of the time period for a specific amount of precipitation to occur. Some examples are:

a. Storage capacity for water management.

b. Reservoir-filling time behind large hydraulic structures.

c. Irrigation and agricultural crop practices.

The determination of this variable requires continuous data.

3. When the functions $\lambda_1(t)$ and $\lambda_2(x)$ were assumed constant, the theoretical distributions were based on λ_1 and λ_2 values that were linear with respect to time. This is not accurate when one considers that these functions are subject to seasonal variations. To accommodate those situations where λ_1 and λ_2 vary rapidly, the theoretical equations could be modified to follow a non-linear relationship with time and, therefore, provide a better agreement between observed and theoretical distributions.

4. The involvement of meteorological theory of airmass circulation and storm movements would be an important asset in the stochastic treatment of the distribution of precipitation as it would provide a means of establishing a classification system for precipitation.

BIBLIOGRAPHY

Canada Meteorological Branch, 1967 - "Temperature and Precipitation Tables for Prairie Provinces" Volume III.

Canada Meteorological Branch, 1970a - "Climatological Station Data Catalogue - The Prairie Provinces".

Canada Meteorological Branch, 1970b - "Tape Format Documentation".

Chow, V.T., 1964 - "Handbook of Applied Hydrology" McGraw-Hill, New York.

Cramer, H., Leadbetter, M.R., 1967 - "Stationary and Related Stochastic Processes" Wiley, New York.

Doob, J.L., 1953 - "Stochastic Processes" Wiley, New York.

Foster, E.E., 1949 - "Rainfall and Runoff" MacMillian, New York.

Linsley, R.K., Franzini, J.B., 1964 - "Water Resource Engineering" McGraw-Hill, New York.

Teweles, S., Giraytys, J., 1970 - "Meteorological Observations and Instrumentation" American Meteorological Society, Boston.

Thom, H.C.S., 1968 - "Direct and Inverse Tables of the Gamma Distribution" Dept. of Commerce, Environmental Science Service Administration, Silver Spring, U.S.A.

Todorovic, P., 1967 - "A Particular Stochastic Process as Applied to Hydrology" Proceedings International Hydrology Symposium, Fort Collins, Colorado.

U.S. Department of the Interior, 1970 - "Project Skywater" Bureau of Reclamation, Technical Report.

Verschuren, J.P., 1968 - "A Stochastic Analysis of Precipitation" Ph.D. Thesis, University of Colorado, Fort Collins, Colorado.

Wiesner, C.J., 1970 - "Hydrometeorology" Chapman and Hall, London.

APPENDIX A

Program for Converting Raw Data for First-order Stations

FORTRAN

```
      DIMENSION STRING(20)
      DATA RAIN/'   T'/
      DATA ZERO/'0000'/
100   FORMAT(20A4)
101   FORMAT(44H                                            ,4A1,
     132H                                )
      READ(4,100) STRING
      WRITE(7,100) STRING
3     READ(4,101,END=2) PCPN
      IF(PCPN.EQ.RAIN) PCPN=ZERO
      WRITE(7,101) PCPN
      GO TO 3
2     ENDFILE 7
      STOP
      END
```

APPENDIX B

Program for Calculating Characteristic Curve Parameters for First-order Stations

FORTRAN

```
      DIMENSION B(31,12,6),C(31,12,6),V(31),NAME(6),ND(12),Z(31)
     1,LMDA1(6),LMDA2(6)
      REAL LMDA1,LMDA2
      DATA ND/31,28,31,30,31,30,31,31,30,31,30,31/
100   FORMAT('4',40X,'PROGRAM FOR FIRST ORDER STATION'//)
101   FORMAT(I7,6X,6A4)
102   FORMAT(44X,F4.2)
103   FORMAT(44X,I7,4X,6A4,//)
104   FORMAT(I4,6(2X,F6.3,2X,F6.3))
105   FORMAT('MNTH',2X,'LMDA1',2X,'LMDA2',4X,'LMDA1',2X,'LMDA2',
     14X,'LMDA1',2X,'LMDA2',4X,'LMDA1,2X,'LMDA2',4X,'LMDA1',2X,
     2'LMDA2',4X,'LMDA1',2X,'LMDA2',/)
      WRITE(6,100)
      READ(7,101) IP,(NAME(I),I=1,6)
      WRITE(6,103) IP,(NAME(I),I=1,6)
      WRITE(6,105)
      IFY=1938
      DO 1 I=1,31
      DO 2 J=1,12
      DO 11 M=1,31
11    Z(M)=0
      KMAX=ND(J)
      IF(J.NE.2) GO TO 4
      IF(MOD(IFY+I-1,4).EQ.0) KMAX=KMAX+1
4     DO 3 K=1,KMAX
      READ(7,102,END=8) V(K)
      IF(V(K).GT.9.99) GO TO 4
      IF(V(K).GT.0.0) Z(K)=1
3     CONTINUE
      JMAX=KMAX/5
      DO 5 K=1,JMAX
      SUMB=0.0
      SUMC=0.0
      DO 6 L=1,5
      SUMB=SUMB+V(5*(K-1)+L)
      SUMC=SUMC+Z(5*(K-1)+L)
6     CONTINUE
      B(I,J,K)=SUMB
      C(I,J,K)=SUMC
5     CONTINUE
2     CONTINUE
1     CONTINUE
```

APPENDIX B (CONT'D)

```
      DO 7 J=1,12
      IF(J.NE.2) JMAX=6
      IF(J.EQ.2) JMAX=5
      DO 9 K=1,JMAX
      SUML1=0.0
      SUML2=0.0
      DO 10 I=1,31
      SUML1=SUML1+B(I,J,K)
      SUML2=SUML2+C(U,J,K)
10    CONTINUE
      LMDA1(K)=SUML2/(5*31)
      LMDA2(K)=SUML2/SUML1
9     CONTINUE
      WRITE(6,104) J,((LMDA1(K),LMDA2(K)),K=1,JMAX)
7     CONTINUE
8     STOP
      END
```

APPENDIX C

Computer Output for First-order Station

PROGRAM FOR FIRST ORDER STATION

3012200 EDMONTON A

MNTH	LMDA1	LMDA2	LMDA1	LMDA2	LMDA1	LMDA2	LMDA1	LMDA2	LMDA1	LMDA2	LMDA1	LMDA2
1	.335	10.70	.342	14.64	.394	10.91	.426	13.44	.361	12.39	.432	10.52
2	.400	12.02	.252	13.78	.374	14.68	.465	13.21	.335	11.58		
3	.335	16.00	.374	15.07	.374	12.80	.277	10.36	.277	14.88	.297	13.86
4	.297	8.19	.265	10.79	.245	8.88	.226	5.87	.284	10.65	.226	5.56
5	.303	4.57	.232	6.38	.232	6.53	.277	5.86	.323	5.71	.303	5.31
6	.329	5.15	.374	4.21	.394	4.49	.510	3.88	.471	4.24	.432	4.30
7	.413	4.82	.413	4.56	.374	5.36	.406	3.47	.445	3.81	.426	3.92
8	.413	2.92	.361	3.96	.413	5.44	.335	4.86	.316	4.08	.413	5.35
9	.303	7.56	.245	6.66	.323	4.19	.355	6.42	.271	9.70	.239	11.75
10	.277	7.85	.174	7.44	.213	8.48	.142	13.33	.239	10.63	.226	7.92
11	.200	8.96	.219	12.10	.368	9.76	.297	16.25	.361	10.65	.258	11.91
12	.290	17.18	.290	15.41	.284	10.86	.361	13.90	.439	14.59	.452	9.94

APPENDIX D

Program for Converting Raw Data for Ordinary Climate Stations

FORTRAN

```
      DIMENSION IY(6),IM(6),PCPN(6),STRING(20),ND(12)
     1ID(6),T1(6,9),T2(6,8)
      DATA ND/31,28,31,30,31,30,31,31,30,31,30,31/
      DATA R,C,T,BLANK/'   L','   C','   T','    '/
      DATA ZERO/'0000'/
      DATA DNEG/'- 99'/
100   FORMAT(20A4)
102   FORMAT(I7,I2,I2,8A4,A1,A4,8A4)
      READ(4,100) STRING
      WRITE(7,100) STRING
28    DO 10 K=1,5
      READ(4,102,END=8) ID(K),IY(K),IM(K),(T1(K,I),I=1,9),
     1PCPN(K),(T2(K,I),I=1,8)
      JK=1
      LM=IM(K)
      LY=IY(K)
      N=ND(LM)
      IF(PCPN(K).GT.9.99) GO TO 28
      IF(LM.NE.2) GO TO 10
      IF(MOD((LY+1900),4).EQ.0) N=N+1
10    CONTINUE
      II=1
31    IF(PCPN(5).EQ.R.OR.PCPN(5).EQ.C) GO TO 23
      JJ=0
24    DO 11 J=1,4
      IF(PCPN(J).EQ.R.OR.PCPN(J).EQ.C.OR.PCPN(J).EQ.BLANK)
     1GO TO 15
11    CONTINUE
      DO 12 M=1,5
      IF(PCPN(M).EQ.T) PCPN(M)=ZERO
12    CONTINUE
21    DO 70 K=1,5
      WRITE(7,102) ID(K),IY(K),IM(K),(T1(K,I),I=1,9),PCPN(K),
     1(T2(K,I),I=1,8)
70    CONTINUE
41    IF(JK.EQ.3) GO TO 28
      II=II+1
      IF(II.EQ.6) GO TO 18
      DO 19 K=1,5
19    READ(4,102) ID(K),IY(K),IM(K),(T1(K,I),I=1,9),PCPN(K),
     1(T2(K,I),I=1,8)
      IF(JJ.EQ.1) GO TO 20
```

APPENDIX D (CONT'D)

```
      GO TO 31
23    JJ=1
15    DO 16 J=1,5
16    PCPN(J)=DNEG
      GO TO 21
20    IF(PCPN(5).EQ.R.OR.PCPN(5).EQ.C) GO TO 25
      JJ=0
26    DO 22 J=1,5
22    PCPN(J)=DNEG
      GO TO 21
25    JJ=1
      GO TO 26
18    IQ=N-27
      GO TO (1,2,3,4),IQ
1     DO 27 K=1,3
27    READ(4,102) ID(K),IY(K),IM(K),(T1(K,I),I=1,9),PCPN(K),
     1(T2(K,I),I=1,8)
      GO TO 28
2     DO 29 K=1,4
29    READ(4,102) ID(K),IY(K),IM(K),(T1(K,I),I=1,9),PCPN(K),
     1(T2(K,I),I=1,8)
      GO TO 28
3     DO 30 K=1,5
30    READ(4,102) ID(K),IY(K),IM(K),(T1(K,I),I=1,9),PCPN(K),
     1(T2(K,I),I=1,8)
      JK=3
      GO TO 31
4     DO 32 K=1,6
32    READ(4,102) ID(K),IY(K),IM(K),(T1(K,I),I=1,9),PCPN(K),
     1(T2(K,I),I=1,8)
      JK=3
      GO TO 31
8     STOP
      END
```

APPENDIX E

Program for Calculating Characteristic Curve Parameters for Ordinary Climatological Stations

FORTRAN

```
      DIMENSION A(31,12,6),B(31,12,6),PCPN(5),NAME(6),
     1LMDA1(6),LMDA2(6),IY(5),IM(5)
      REAL LMDA1,LMDA2
100   FORMAT('4',45X,'PROGRAM FOR ORDINARY CLIMATE STATION'//)
101   FORMAT(I7,6X,6A4)
102   FORMAT(7X,2I2,33X,F4.2)
103   FORMAT(50X,I7,4X,6A4,//)
104   FORMAT(I4,6(2X,F6.3,2X,F6.3))
105   FORMAT('MNTH',2X,'LMDA1',2X,'LMDA2',4X,'LMDA1',2X,'LMDA2',
     14X,'LMDA1',2X,'LMDA2',4X,'LMDA1',2X,'LMDA2',4X,'LMDA1',2X,
     2'LMDA2',4X,'LMDA1',2X,'LMDA2'/)
      WRITE(6,100)
      READ(7,101) IP,(NAME(I),I=1,6)
      WRITE(6,103) IP,(NAME(I),I=1,6)
      WRITE(6,105)
      IFY=26
      IFM=1
      II=1
      IPY=1
      Z=0.0
4     DO 14 K=1,5
      READ(7,102,END=8) IY(K),IM(K),PCPN(K)
      IF(PCPN(K).GT.0.0) Z=Z+1
14    CONTINUE
34    IF(IM(1).NE.IFM) GO TO 9
      IF(IY(1).NE.IFY) GO TO 36
      SUMA=0.0
      DO 5 K=1,5
      SUMA=SUMA+PCPN(K)
5     CONTINUE
      A(IPY,IFM,II)=SUMA
      B(IPY,IFM,II)=Z
      II=II+1
      Z=0.0
      IF(IFM.EQ.2.AND.II.EQ.6) GO TO 19
      IF(II.EQ.7) GO TO 19
      GO TO 4
19    IFM=IFM+1
      II=1
      Z=0.0
      IF(IFM.GT.12) GO TO 16
      GO TO 4
16    IFY=IFY+1
```

APPENDIX E (CONT'D)

```
      IPY=IPY+1
      IFM=1
      II=1
      Z=0.0
      IF(IFY.GT.56) GO TO 18
      GO TO 4
36    DO 13 IFM=1,12
      DO 23 II=1,6
      A(IPY,IFM,II)=-.99
      B(IPY,IFM,II)=-.99
23    CONTINUE
13    CONTINUE
44    IFY=IFY+1
      IPY=IPY+1
      IFM=1
      II=1
      Z=0.0
      IF(IFY.GT.56) GO TO 18
      GO TO 34
9     DO 33 II=1,6
      A(IPY,IFM,II)=-.99
      B(IPY,IFM,II)=-.99
33    CONTINUE
      IFM=IFM+1
      II=1
      Z=0.0
      IF(IFM.GT.12) GO TO 44
      GO TO 34
18    DO 7 IFM=1,12
      IF(IFM.NE.2) IMAX=6
      IF(IFM.EQ.2) IMAX=5
      DO 17 II=1,IMAX
      COUNT=0.0
      SUML1=0.0
      SUML2=0.0
      DO 21 IPY=1,31
      IF(A(IPY,IFM,II).LT.0.0) GO TO 21
      SUML1=SUML1+A(IPY,IFM,II)
      SUML2=SUML2+B(IPY,IFM,II)
      COUNT=COUNT+1
21    CONTINUE
      LMDA1(II)=SUML2/(5.0*COUNT)
      LMDA2(II)=SUML2/SUML1
17    CONTINUE
      WRITE(6,104) IFM,((LMDA1(II),LMDA2(II)),II=1,IMAX)
7     CONTINUE
8     STOP
      END
```

APPENDIX F

Program for Solution of Distribution Function of X_t

FORTRAN

```
C       PROGRAM XSUBT
        DIMENSION T(6),FF(100),FFF(100)
        LAMDAS=1
        DX=0.05
        READ(5,4) T
4       FORMAT(6F10.1)
        DO 50 M=1,LAMDAS
        READ(5,5) AL1,AL2
5       FORMAT(2F10.3)
        DO 50 K=1,6
        FTX2=0.0
        WRITE(6,51) AL1,AL2,T(K)
51      FORMAT(1H1,'LAMDA1 = ',F5.3,'LAMDA2 = ',F5.2,' T = ',F5.1)
        WRITE(6,52)
52      FORMAT(1H,'      X     DIST. FUNT.     INCREMENTS')
        X=0.0
        A=AL1*T(K)
        FTX1=EXP(-A)
        SUMX=FTX1
        GO TO 54
3       B=AL2*X
        SUML=0.0
        DO 60 J=1,50
        AJ=J
        SUML=SUML+ALOG(AJ)
        FFL=-B+J*ALOG(B)-SUML
        FF(J)=EXP(FFL)
        FFFL=-A+J*ALOG(A)-SUML
60      FFF(J)=EXP(FFFL)
        SUM1=EXP(-B)
        DO 12 J=1,50
12      SUM1=SUM1+FF(J)
        SUMX=SUM1*EXP(-A)
        DO 15 L=1,50
        SUM=0.0
        DO 14 J=L,50
14      SUM=SUM+FF(J)
        SUM=SUM*FFF(L)
15      SUMX=SUMX+SUM
        FTX1=SUMX
        IF(ABS(1.0-FTX1).LT.0.001) GO TO 50
54      DFTX=FTX1-FTX2
        IF(X.EQ.0.05) DFTX=FTX1
```

APPENDX F (CONT'D)

```
      WRITE(6,53) X,FTX1,DFTX
53    FORMAT(5X,F5.2,6X,F9.3,6X,F9.3)
      FTX2=FTX1
      X=X+DX
50    CONTINUE
      STOP
      END
```

APPENDIX G

Meteorological Information of the Selected Stations

TABLE I

SELECTED PRECIPITATION STATIONS OF ALBERTA

Station Name	Lat. N	Long. W	Elevation†	Years of Record††
Cardston	49°12'	113°19'	3775	1926 - 1956
Carway	49°00'	113°22'	4460	1926 - 1956
Coleman	49°38'	114°35'	4400	1926 - 1956
Edmonton	53°34'	113°31'	2219	1938 - 1968
Edson	53°35'	116°25'	3033	1929 - 1959
Entrance	53°22'	117°42'	3300	1924 - 1954
Fort MacLeod	49°43'	113°24'	3116	1926 - 1956
Jasper	52°53'	118°04'	3480	1937 - 1967
Lethbridge	49°38'	112°48'	3018	1938 - 1968
Lundbreck	49°55'	114°08'	3918	1926 - 1956
Medicine Hat	50°01'	110°43'	2250	1926 - 1956
Seven Persons	49°50'	110°54'	2480	1926 - 1956
Winnifred	49°54'	111°12'	2725	1926 - 1956

† Present elevation of the station

†† Period of record chosen for this study

APPENDIX G (CONT'D)

TABLE II

PRECIPITATION DATA FOR THE SELECTED STATIONS OF ALBERTA

Station Name	Annual Precipitation	Annual Rainfall	Annual Snowfall	No. of Days With Rain	No. of Days With Snow
Cardston	18.04	11.50	65.4	47	47
Carway	20.37	10.86	95.1	43	43
Coleman	19.91	11.21	87.0	46	34
Edmonton	18.64	13.26	53.8	68	57
Edson	20.85	14.98	58.7	69	38
Entrance	19.98	14.41	55.7	54	39
Fort MacLeod	17.67	12.38	52.9	53	35
Jasper	15.98	11.06	49.2	81	48
Lethbridge	17.23	10.66	65.7	54	56
Lundbreck	19.28	10.99	82.9	37	43
Medicine Hat	14.29	9.42	48.7	54	41
Seven Persons	13.79	9.03	47.6	38	26
Winnifred	13.86	8.92	49.4	45	34

Note: The values presented in this table are based on the period of observation from 1930 to 1960

APPENDIX H

Computational Method for the Lambda Calculations

TABLE III

COMPUTATIONAL METHOD USED TO DERIVE THE λ_1 VALUES

Station Carway, Alberta

Month January

Number of Storms

Period (Days)

Year	1 - 5	6 - 10	11 - 15	16 - 20	21 - 25	26 - 30
1926	0	3	0	etc		
1927	2	2	etc			
'	'	'				
1956	1	etc				
Total	28	etc				

$E(\eta_t) = 28/29 = .966$ etc

$\lambda_1 = \dfrac{E(\eta_t)}{t} = .966/5 = .193$ etc

where 29 is equal to the number of years of record and t is the interval length.

APPENDIX H (CONT'D)

TABLE IV

COMPUTATIONAL METHOD USED TO DERIVE THE λ_2 VALUES

Station Carway, Alberta

Month January

Total Amount of Precipitation

Period (Days)

Year	1 - 5	6 - 10	11 - 15	16 - 20	21 - 25	26 - 30
1926	0	.32	0	etc		
1927	.50	.16	etc			
'	'	'				
1956	.06	etc				
Total	4.85	etc				

$E(X_t) = 4.85/29 = .167$ etc

$$\lambda_2 = \frac{\lambda_1 t}{E(X_t)} = \frac{.193 \times 5}{.167} = 5.78 \quad \text{etc}$$

where 29 is equal to the number of years of record and t is the interval length.

APPENDIX I

Comparisons Between the Observed and Theoretical Distributions

TABLE VII

EDMONTON - NO. OF DAYS WITH PRECIPITATION DURING 5-DAY PERIODS

No. of days with precipitation		0	1	2	3	4	5
Jan. 1-5	$\lambda_1 = 0.388$						
Obsvd. freq.		8	6	10	3	3	1
Theor. freq.		5	9	8	5	3	1
Feb. 1-5	$\lambda_1 = 0.383$						
Obsvd. freq.		5	4	12	6	4	0
Theor. freq.		5	9	8	5	3	1
Mar. 1-5	$\lambda_1 = 0.360$						
Obsvd. freq.		5	8	13	3	1	1
Theor. freq.		5	9	9	5	2	1
Apr. 1-5	$\lambda_1 = 0.275$						
Obsvd. freq.		10	6	6	8	1	0
Theor. freq.		8	11	7	4	1	0
May 1-5	$\lambda_1 = 0.258$						
Obsvd. freq.		8	8	9	4	1	1
Theor. freq.		9	11	7	3	1	0
June 1-5	$\lambda_1 = 0.335$						
Obsvd. freq.		4	10	12	3	2	0
Theor. freq.		6	10	8	4	2	1
July 1-5	$\lambda_1 = 0.440$						
Obsvd. freq.		3	8	8	8	4	0
Theor. freq.		3	8	8	6	4	2
Aug. 1-5	$\lambda_1 = 0.395$						
Obsvd. freq.		4	5	8	13	1	0
Theor. freq.		4	9	8	6	3	1
Sept. 1-5	$\lambda_1 = 0.332$						
Obsvd. freq.		5	15	4	4	3	0
Theor. freq.		6	10	8	4	2	1
Oct. 1-5	$\lambda_1 = 0.246$						
Obsvd. freq.		8	8	12	1	2	0
Theor. freq.		9	11	7	3	1	0
Nov. 1-5	$\lambda_1 = 0.235$						
Obsvd. freq.		13	9	6	2	1	0
Theor. freq.		10	11	7	2	1	0
Dec. 1-5	$\lambda_1 = 0.318$						
Obsvd. freq.		8	7	12	2	2	0
Theor. freq.		6	10	8	4	2	1

TABLE VIII

CENTRAL ALBERTA STATIONS - NO. OF DAYS WITH PRECIPITATION DURING 5-DAY INTERVAL JAN. 1-5

No. of days with precipitation		0	1	2	3	4	5
Edmonton	λ_1 = 0.388						
Obsvd. freq.		8	6	10	3	3	1
Theor. freq.		5	9	8	5	3	1
Edson	λ_1 = 0.212						
Obsvd. freq.		16	7	5	2	1	0
Theor. freq.		11	11	6	2	1	0
Entrance	λ_1 = 0.213						
Obsvd. freq.		14	6	5	3	2	0
Theor. freq.		10	11	6	2	1	0
Jasper	λ_1 = 0.350						
Obsvd. freq.		4	9	6	8	2	1
Theor. freq.		5	9	8	5	2	1

TABLE IX

EDMONTON AND EDSON - NO. OF DAYS WITH PRECIPITATION - PERIOD JUNE 1-20

No. of days with precipitation	Edmonton $\lambda_1 = 0.381$ Obsvd. freq.	Theor. freq.	Edson $\lambda_1 = 0.421$ Obsvd. freq.	Theor. freq.
0	0	0	0	0
1	0	0	0	0
2	1	0	0	0
3	2	1	2	1
4	1	2	2	2
5	3	3	1	2
6	3	4	5	3
7	4	5	2	4
8	1	4	5	4
9	4	4	2	4
10	4	3	1	3
11	4	2	5	3
12	4	1	3	2
13	0	1	2	1
14-20	0	1	1	2

TABLE X

ENTRANCE AND JASPER - NO. OF DAYS WITH PRECIPITATION - PERIOD MAY 21 - JUNE 10

No. of days with precipitation	Entrance $\lambda_1 = 0.354$ Obsvd. freq.	Theor. freq.	Jasper $\lambda_1 = 0.349$ Obsvd. freq.	Theor. freq.
0	0	0	0	0
1	0	0	0	0
2	1	1	0	1
3	2	1	2	2
4	3	3	2	3
5	1	3	7	4
6	5	4	3	5
7	3	4	4	5
8	4	4	3	4
9	2	3	3	3
10	3	2	2	2
11	1	1	4	1
12	1	1	0	1
13	1	1	0	0
14-20	1	0	1	0

TABLE XI

JASPER AND EDSON - NO. OF DAYS WITH PRECIPITATION - PERIOD JAN. 1-30

No. of days with precipitation	Jasper $\lambda_1 = 0.350$		Edson $\lambda_1 = 0.212$	
	Obsvd. freq.	Theor. freq.	Obsvd. freq.	Theor. freq.
0-1	0	0	1	0
2-3	1	0	8	3
4-5	3	1	3	8
6-7	5	4	6	10
8-9	5	6	6	6
10-11	3	7	6	3
12-13	4	6	0	1
14-15	3	3	1	0
16-17	5	2	0	0
18-19	1	1	0	0
20-30	0	0	0	0

TABLE XII

EDMONTON AND ENTRANCE - NO. OF DAYS WITH PRECIPITATION - PERIOD JAN. 1-30

No. of days with precipitation	Edmonton $\lambda_1 = 0.385$		Entrance $\lambda_1 = 0.213$	
	Obsvd. freq.	Theor. freq.	Obsvd. freq.	Theor. freq.
0-1	0	0	2	0
2-3	1	0	8	3
4-5	2	1	6	8
6-7	3	3	2	9
8-9	4	5	6	6
1o-11	6	7	4	3
12-13	5	7	0	1
14-15	4	5	0	0
16-17	4	2	0	0
18-19	2	1	2	0
20-30	0	0	0	0

TABLE XIII

CENTRAL ALBERTA STATIONS - AMOUNT OF PRECIPITATION DURING 5 DAYS

t_o = 0 hrs. Jan. 1st.

frequencies

Amount of precipitation	Edmonton Obsvd.	Edmonton Theor.	Edson Obsvd.	Edson Theor.	Entrance Obsvd.	Entrance Theor.	Jasper Obsvd.	Jasper Theor.
0	8	4	16	11	14	10	4	5
0 - 0.20	12	17	8	11	11	12	16	13
0.21 - 0.40	9	7	2	5	3	5	5	7
0.41 - 0.60	1	2	2	2	0	2	0	3
0.61 - 0.80	1	1	1	1	0	1	2	1
0.81 - 1.00	0	0	0	1	1	0	2	1
1.01 - 1.20	0	0	2	0	0	0	0	0
1.21 - 1.40	0	0	0	0	1	0	0	0
1.41 - ∞	0	0	0	0	0	0	1	0

TABLE XIV

CENTRAL ALBERTA STATIONS - AMOUNT OF PRECIPITATION DURING 10 DAYS

t_o = 0 hrs. Jan. 1st

frequencies

Amount of precipitation	Edmonton Obsvd.	Edmonton Theor.	Edson Obsvd.	Edson Theor.	Entrance Obsvd.	Entrance Theor.	Jasper Obsvd.	Jasper Theor.
0	1	1	6	4	11	4	1	1
0 - 0.30	18	16	16	14	13	16	13	12
0.31 - 0.60	10	10	6	8	2	7	8	10
0.61 - 0.90	1	3	1	4	1	2	1	5
0.91 - 1.20	0	1	1	1	1	1	4	2
1.21 - 1.50	1	0	1	0	2	0	1	0
1.51 - 1.80	0	0	0	0	0	0	1	0
1.81 - 2.10	0	0	0	0	0	0	0	0
2.11 - ∞	0	0	0	0	0	0	1	0

TABLE XV

CENTRAL ALBERTA STATIONS - AMOUNT OF PRECIPITATION DURING 15 DAYS

t_o = 0 hrs. Jan. 1st.

Amount of precipitation	Edmonton Obsvd.	Edmonton Theor.	Edson Obsvd.	Edson Theor.	Entrance Obsvd.	Entrance Theor.	Jasper Obsvd.	Jasper Theor.
	frequencies							
0	0	0	3	1	5	1	0	0
0 - 0.40	16	14	14	14	17	16	13	10
0.41 - 0.80	11	13	9	10	4	9	5	12
0.81 - 1.20	3	3	3	4	1	3	7	6
1.21 - 1.60	1	1	2	2	1	1	3	2
1.61 - 2.00	0	0	0	0	1	0	1	0
2.01 - 2.40	0	0	0	0	1	0	0	0
2.41 - 2.80	0	0	0	0	0	0	0	0
2.81 - ∞	0	0	0	0	0	0	1	0

TABLE XVI

CENTRAL ALBERTA STATIONS - AMOUNT OF PRECIPITATION DURING 20 DAYS

t_o = 0 hrs. Jan. 1st.

Amount of precipitation	Edmonton Obsvd.	Edmonton theor.	Edson Obsvd.	Edson Theor.	Entrance Obsvd.	Entrance Theor.	Jasper Obsvd.	Jasper Theor.
	frequencies							
0	0	0	0	0	1	0	0	0
0 - 0.40	11	8	15	10	15	12	8	5
0.41 - 0.80	13	15	5	11	7	11	6	11
0.81 - 1.20	6	6	8	6	4	5	6	9
1.21 - 1.60	0	2	2	3	1	1	8	4
1.61 - 2.00	1	0	1	1	1	1	1	1
2.01 - 2.40	0	0	0	0	1	0	0	0
2.41 - 2.80	0	0	0	0	0	0	0	0
2.81 - 3.20	0	0	0	0	0	0	0	0
3.21 - ∞	0	0	0	0	0	0	0	0

TABLE XVII

CENTRAL ALBERTA STATIONS - AMOUNT OF PRECIPITATION DURING 25 DAYS

t_o = 0 hrs. Jan. 1st.

Amount of precipitation	frequencies							
	Edmonton		Edson		Entrance		Jasper	
	Obsvd.	Theor.	Obsvd.	Theor.	Obsvd.	Theor.	Obsvd.	Theor.
0	0	0	0	0	1	0	0	0
0 - 0.50	11	7	14	9	11	12	9	4
0.51 - 1.00	11	15	6	12	10	12	8	12
1.01 - 1.50	8	6	6	7	4	5	9	9
1.51 - 2.00	1	2	4	2	2	1	2	4
2.01 - 2.50	0	0	0	1	2	0	1	1
2.51 - 3.00	0	0	1	0	0	0	0	0
3.01 - 3.50	0	0	0	0	0	0	0	0
3.51 - ∞	0	0	0	0	0	0	1	0

TABLE XVIII

CENTRAL ALBERTA STATIONS - AMOUNT OF PRECIPITATION DURING 30 DAYS

t_o = 0 hrs. Jan. 1st.

Amount of precipitation	frequencies							
	Edmonton		Edson		Entrance		Jasper	
	Obsvd.	Theor.	Obsvd.	Theor.	Obsvd.	Theor.	Obsvd.	Theor.
0	0	0	0	0	1	0	0	0
0 - 0.50	5	4	8	6	11	8	5	2
0.51 - 1.00	13	15	10	11	9	13	12	9
1.01 - 1.50	9	9	7	8	5	6	3	10
1.51 - 2.00	4	3	4	4	2	2	8	6
2.01 - 2.50	0	0	0	1	1	1	1	2
2.51 - 3.00	0	0	2	1	1	0	0	1
3.01 - 3.50	0	0	0	0	0	0	0	0
3.51 - 4.00	0	0	0	0	0	0	0	0
4.01 - ∞	0	0	0	0	0	0	1	0

TABLE XIX

MEDICINE HAT AND SEVEN PERSONS - AMOUNT OF PRECIPITATION DURING 5 DAYS

t_o = 0 hrs. Aug. 1st.

Amount of precipitation	Medicine Hat Obsvd. freq.	Medicine Hat Theor. freq.	Seven Persons Obsvd. freq.	Seven Persons Theor. freq.
0	13	9	19	15
0 - 0.20	14	10	4	6
0.21 - 0.40	1	6	4	4
0.41 - 0.60	1	3	0	2
0.61 - 0.80	0	2	2	2
0.81 - 1.00	0	1	1	1
1.01 - 1.20	1	0	1	1
1.21 - 1.40	1	0	0	0
1.41 - ∞	0	0	0	0

(frequencies)

TABLE XX

MEDICINE HAT AND SEVEN PERSONS - AMOUNT OF PRECIPITATION DURING 10 DAYS

t_o = 0 hrs. Aug. 1st.

Amount of precipitation	Medicine Hat Obsvd. freq.	Medicine Hat Theor. freq.	Seven Persons Obsvd. freq.	Seven Persons Theor. freq.
0	4	3	10	7
0 - 0.30	17	11	8	10
0.31 - 0.60	4	7	3	6
0.61 - 0.90	1	5	5	4
0.91 - 1.20	1	3	2	2
1.21 - 1.50	0	1	1	1
1.51 - 1.80	2	1	2	1
1.81 - 2.10	2	0	0	0
2.11 - ∞	0	0	0	0

(frequencies)

TABLE XXI

MEDICINE HAT AND SEVEN PERSONS - AMOUNT OF PRECIPITATION DURING 15 DAYS

t_o = 0 hrs. Aug. 1st.

Amount of precipitation	Medicine Hat Obsvd. freq.	Medicine Hat Theor. freq.	Seven Persons Obsvd. freq.	Seven Persons Theor. freq.
0	3	1	7	3
0 - 0.40	14	10	8	11
0.41 - 0.80	3	10	5	8
0.81 - 1.20	4	6	4	5
1.21 - 1.60	2	3	4	2
1.61 - 2.00	1	1	2	1
2.01 - 2.40	2	0	0	1
2.41 - 2.80	1	0	0	0
2.81 - ∞	1	0	1	0

TABLE XXII

MEDICINE HAT AND SEVEN PERSONS - AMOUNT OF PRECIPITATION DURING 20 DAYS

t_o = 0 hrs. Aug. 1st.

Amount of precipitation	Medicine Hat Obsvd. freq.	Medicine Hat Theor. freq.	Seven Persons Obsvd. freq.	Seven Persons Theor. freq.
0	1	0	6	2
0 - 0.40	11	6	7	8
0.41 - 0.80	6	9	3	8
0.81 - 1.20	4	8	5	6
1.21 - 1.60	4	5	4	3
1.61 - 2.00	1	2	5	2
2.01 - 2.40	2	1	0	1
2.41 - 2.80	0	0	0	1
2.81 - 3.20	1	0	0	0
3.21 - ∞	1	0	1	0

TABLE XXIII

MEDICINE HAT AND SEVEN PERSONS - AMOUNT OF PRECIPITATION DURING 25 DAYS

t_o = 0 hrs. Aug. 1st.

Amount of precipitation	Medicine Hat Obsvd. freq.	Medicine Hat Theor. freq.	Seven Persons Obsvd. freq.	Seven Persons Theor. freq.
0	1	0	4	1
0 - 0.50	10	5	6	8
0.51 - 1.00	5	10	6	9
1.01 - 1.50	5	8	2	6
1.51 - 2.00	2	5	8	4
2.01 - 2.50	4	2	3	2
2.51 - 3.00	2	1	1	1
3.01 - 3.50	1	0	0	0
3.51 - ∞	1	0	1	0

TABLE XXIV

MEDICINE HAT AND SEVEN PERSONS - AMOUNT OF PRECIPITATION DURING 30 DAYS

t_o = 0 hrs. Aug. 1st.

Amount of precipitation	Medicine Hat Obsvd. freq.	Medicine Hat Theor. freq.	Seven Persons Obsvd. freq.	Seven Persons Theor. freq.
0	0	0	3	0
0 - 0.50	10	3	6	6
0.51 - 1.00	3	8	4	8
1.01 - 1.50	5	9	2	7
1.51 - 2.00	3	6	8	5
2.01 - 2.50	2	3	4	3
2.51 - 3.00	5	1	0	1
3.01 - 3.50	1	1	2	1
3.51 - 4.00	0	0	2	0
4.01 - ∞	2	0	0	0

TABLE XXV

AMOUNT OF PRECIPITATION DURING 5 DAYS FOR CENTRAL ALBERTA

Average frequencies of the central Alberta stations combined

t_o = 0 hrs. Jan. 1st.

Amount of precipitation	Obsvd. ave.	Theor. ave.$_1$	Theor. ave.$_2$
0	10	8	7
0 - 0.20	12	13	14
0.21 - 0.40	5	6	6
0.41 - 0.60	1	2	2
0.61 - 0.80	1	1	1
0.81 - 1.00	1	1	1
1.01 - 1.20	1	0	0
1.21 - 1.40	0	0	0
1.41 - ∞	0	0	0

TABLE XXVI

AMOUNT OF PRECIPITATION DURING 10 DAYS FOR CENTRAL ALBERTA

Average frequencies of the central Alberta stations combined

t_o = 0 hrs. Jan. 1st.

Amount of precipitation	Obsvd. ave.	Theor. ave.$_1$	Theor. ave.$_2$
0	5	3	2
0 - 0.30	15	15	15
0.31 - 0.60	7	9	9
0.61 - 0.90	1	3	4
0.91 - 1.20	2	1	1
1.21 - 1.50	1	0	0
1.51 - 1.80	0	0	0
1.81 - 2.10	0	0	0
2.01 - ∞	0	0	0

TABLE XXVII

AMOUNT OF PRECIPITATION DURING 15 DAYS FOR CENTRAL ALBERTA

Average frequencies of the central Alberta stations combined

t_o = 0 hrs. Jan. 1st.

Amount of precipitation	Obsvd. ave.	Theor. ave.$_1$	Theor. ave.$_2$
0	2	1	0
0 - 0.40	15	14	14
0.41 - 0.80	7	11	12
0.81 - 1.20	4	4	4
1.21 - 1.60	2	1	1
1.61 - 2.00	1	0	0
2.01 - 2.40	0	0	0
2.41 - 2.80	0	0	0
2.81 - ∞	0	0	0

TABLE XXVIII

AMOUNT OF PRECIPITATION DURING 20 DAYS FOR CENTRAL ALBERTA

Average frequencies of the central Alberta stations combined

t_o = 0 hrs. Jan. 1st.

Amount of precipitation	Obsvd. ave.	Theor. ave.$_1$	Theor. ave.$_2$
0	0	0	0
0 - 0.40	12	9	8
0.41 - 0.80	8	12	13
0.81 - 1.20	6	6	7
1.21 - 1.60	3	2	2
1.61 - 2.00	1	1	1
2.01 - 2.40	0	0	0
2.41 - 2.80	0	0	0
2.81 - 3.20	0	0	0
3.21 - ∞	0	0	0

TABLE XXIX

AMOUNT OF PRECIPITATION DURING 25 DAYS FOR CENTRAL ALBERTA

Average frequencies of the central Alberta stations combined

t_o = 0 hrs. Jan. 1st.

Amount of precipitation	Obsvd. ave.	Theor. ave.$_1$	Theor. ave.$_2$
0	0	0	0
0 - 0.50	11	8	7
0.51 - 1.00	9	13	14
1.01 - 1.50	7	7	7
1.51 - 2.00	2	2	2
2.01 - 2.50	1	1	0
2.51 - 3.00	0	0	0
3.01 - 3.50	0	0	0
3.51 - ∞	0	0	0

TABLE XXX

AMOUNT OF PRECIPITATION DURING 30 DAYS FOR CENTRAL ALBERTA

Average frequencies of the central Alberta stations combined

t_o = 0 hrs. Jan. 1st.

Amount of precipitation	Obsvd. ave.	Theor. ave.$_1$	Theor. ave.$_2$
0	0	0	0
0 - 0.50	7	5	4
0.51 - 1.00	11	12	13
1.01 - 1.50	6	8	9
1.51 - 2.00	5	4	4
2.01 - 2.50	1	1	1
2.51 - 3.00	1	1	0
3.01 - 3.50	0	0	0
3.51 - ∞	0	0	0

APPENDIX J

Figures

Fig. 4. MAP OF ALBERTA SHOWING PAST AND PRESENT FIRST-ORDER STATIONS AND INDICATING YEARS OF OPERATION

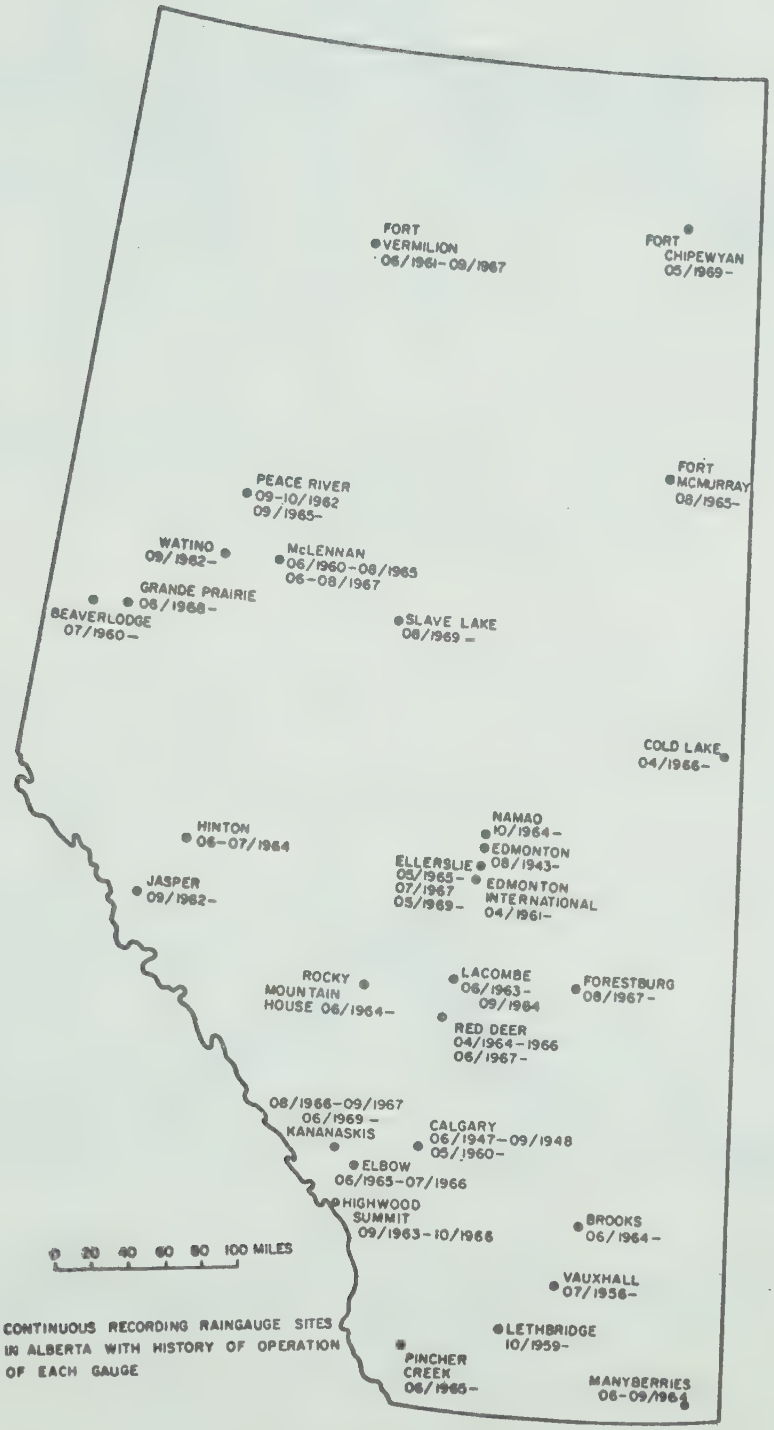

Fig.5. CONTINUOUS RECORDING RAINGAUGE SITES IN ALBERTA WITH HISTORY OF OPERATION OF EACH GAUGE

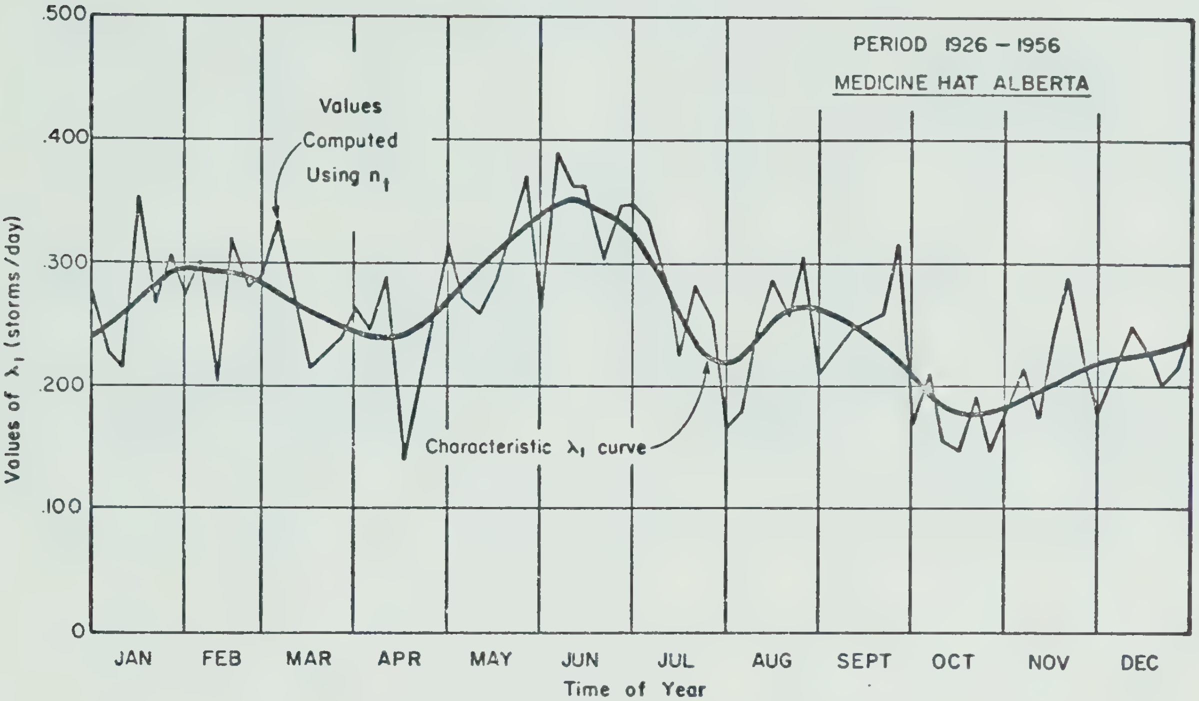

Fig. 6. λ_1 values computed using distribution of n_t

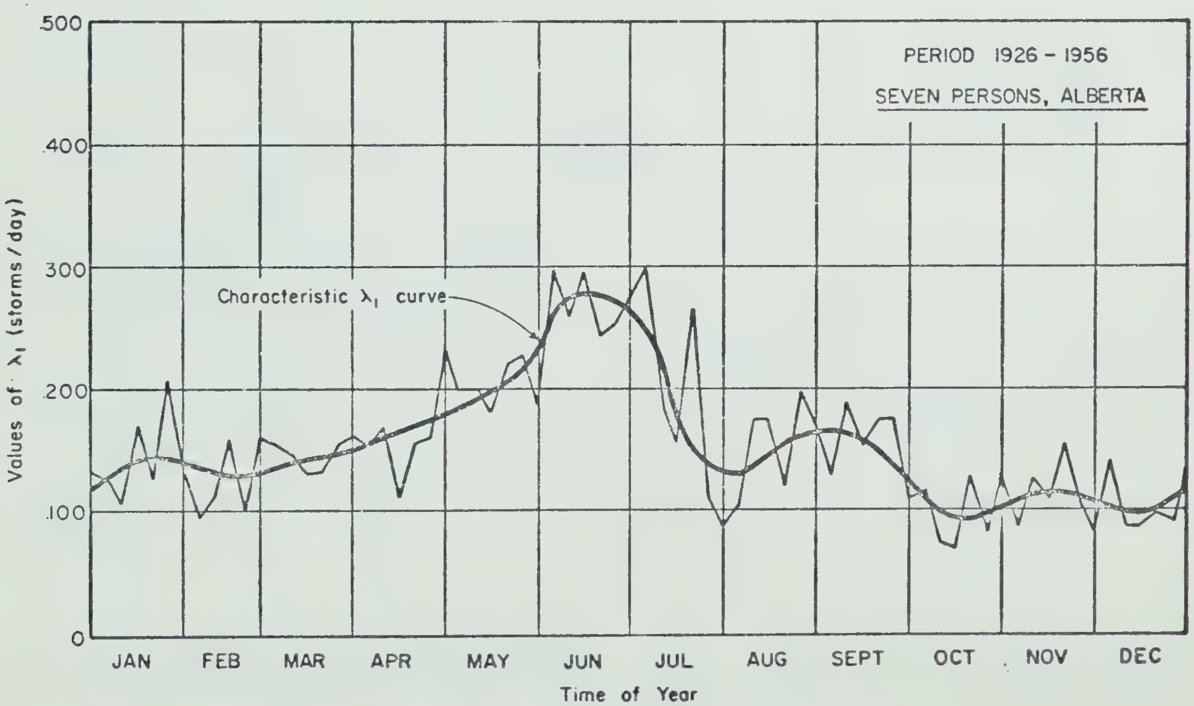

Fig. 7. λ_1 values computed using distribution of n_t

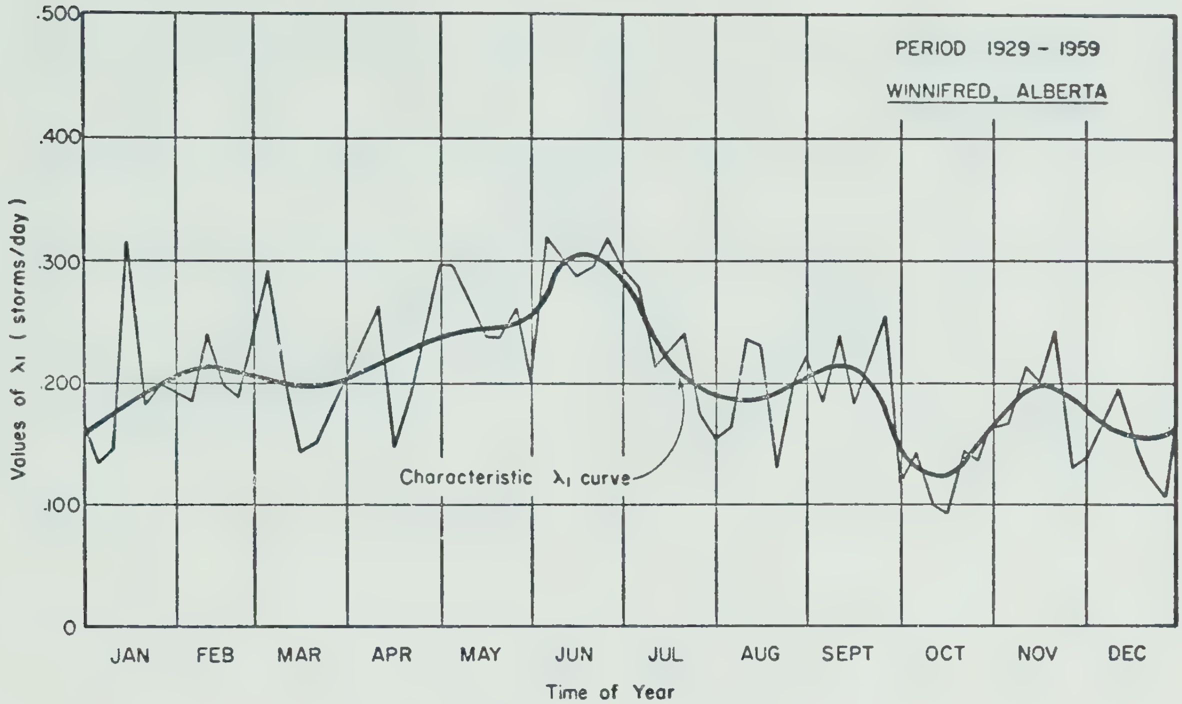

Fig. 8. λ_1 values computed using distribution of n_t

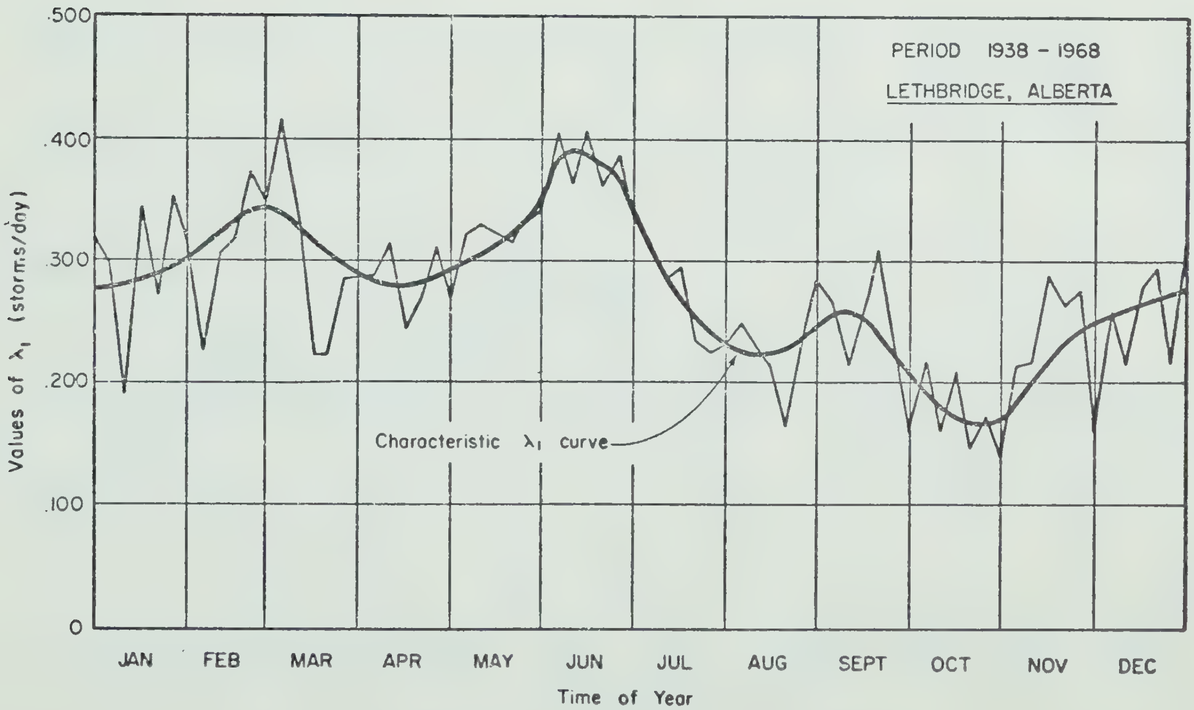

Fig. 9. λ_1 values computed using distribution of n_t

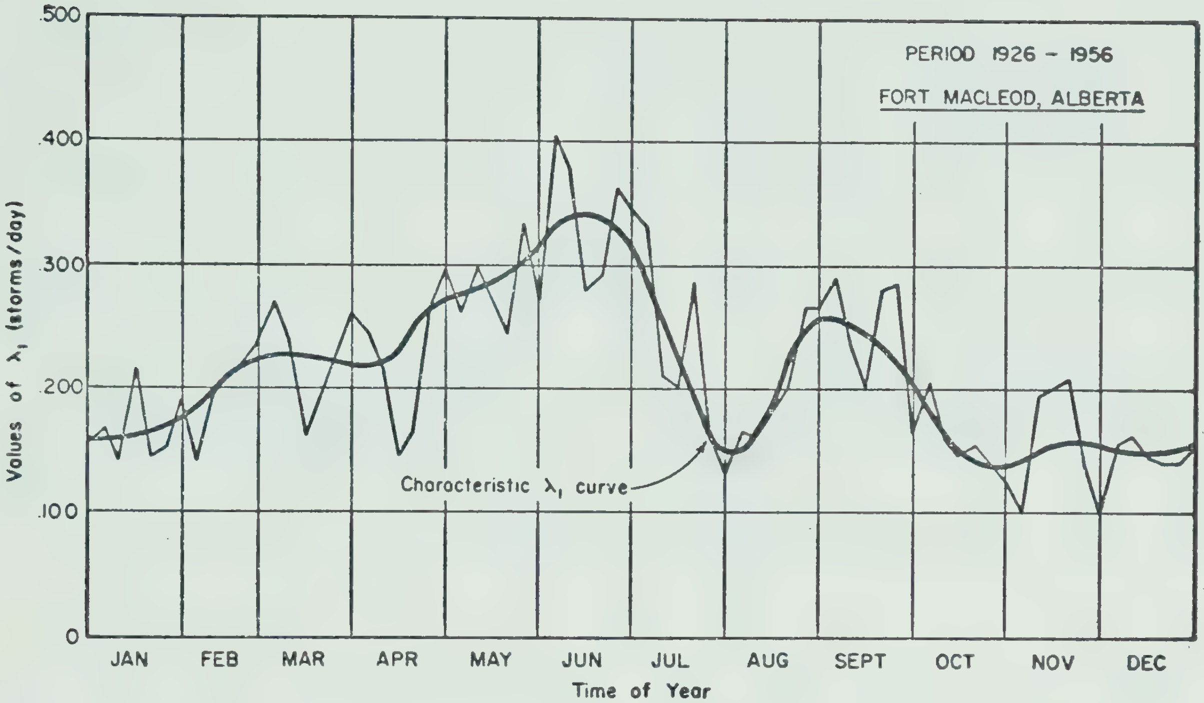

Fig. 10. λ_1 values computed using distribution of n_t

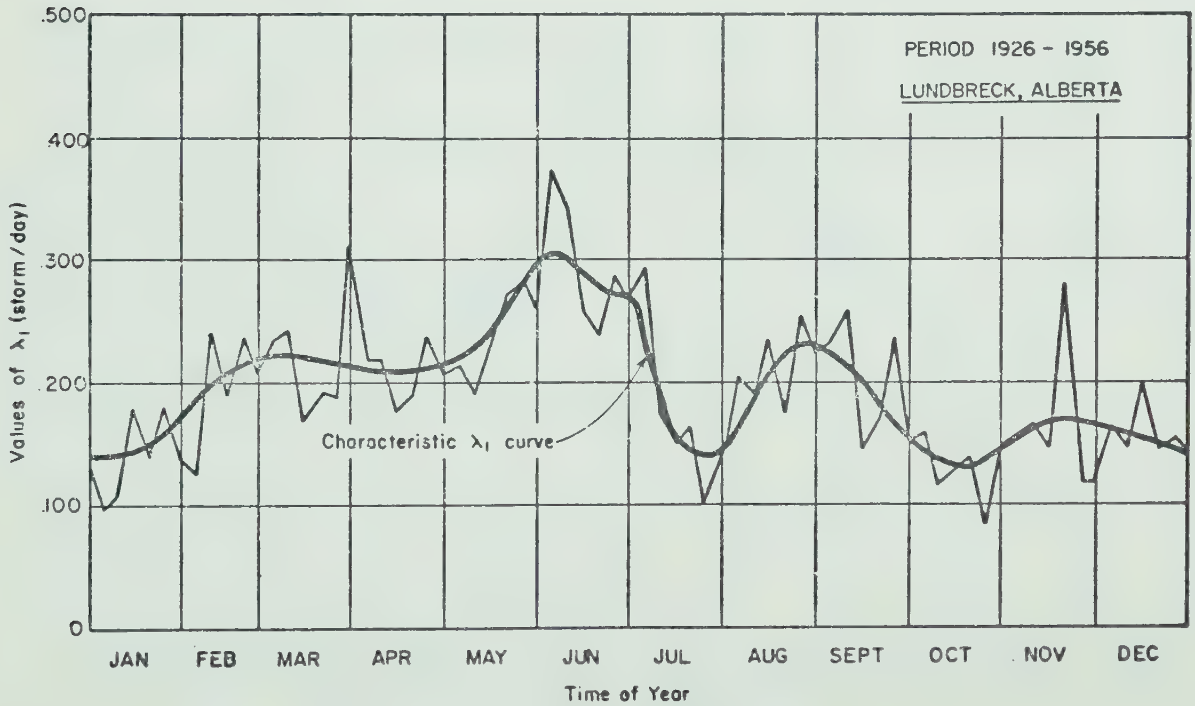

Fig. 11. λ_1 values computed using distribution of n_t

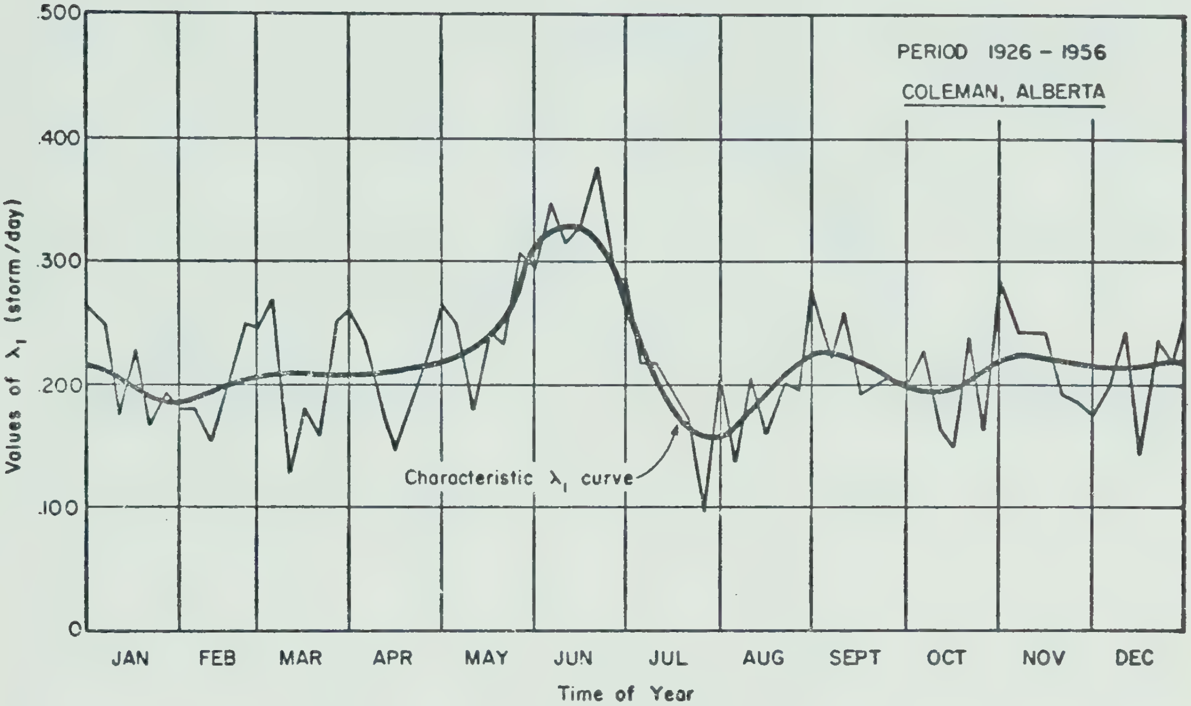

Fig.12. λ_1 values computed using distribution of n_t.

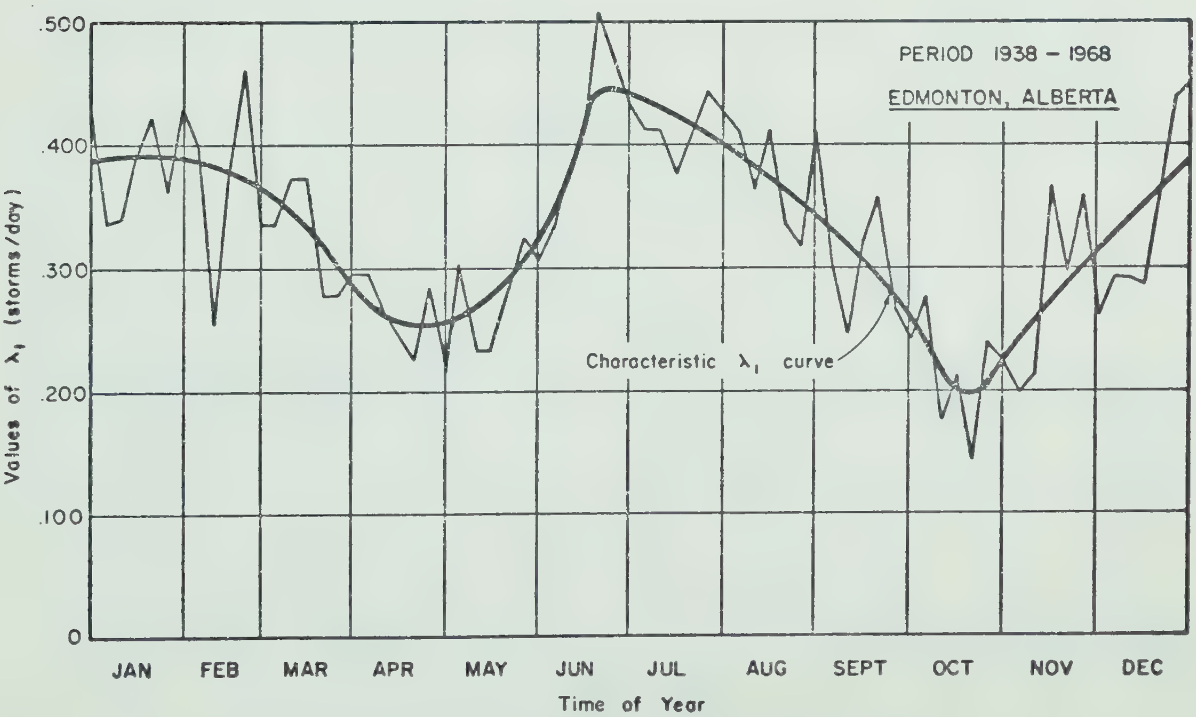

Fig.13. λ_1 values computed using distribution of n_t

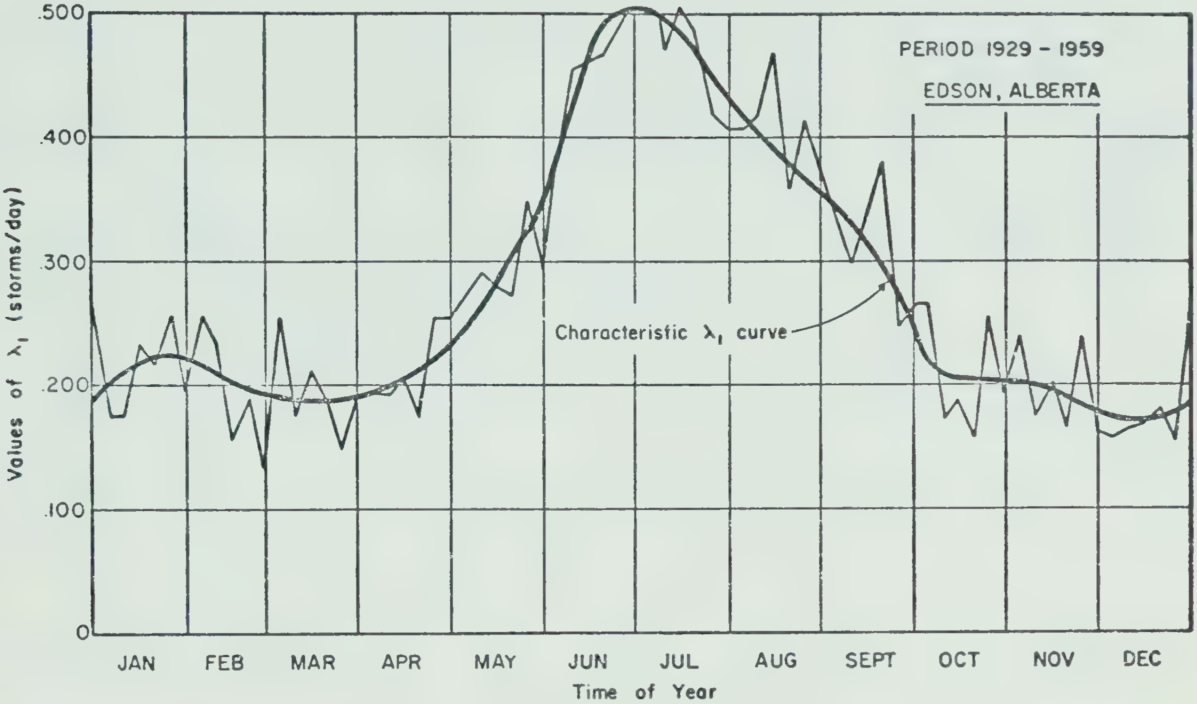

Fig. 14. λ_1 values computed using distribution of n_t

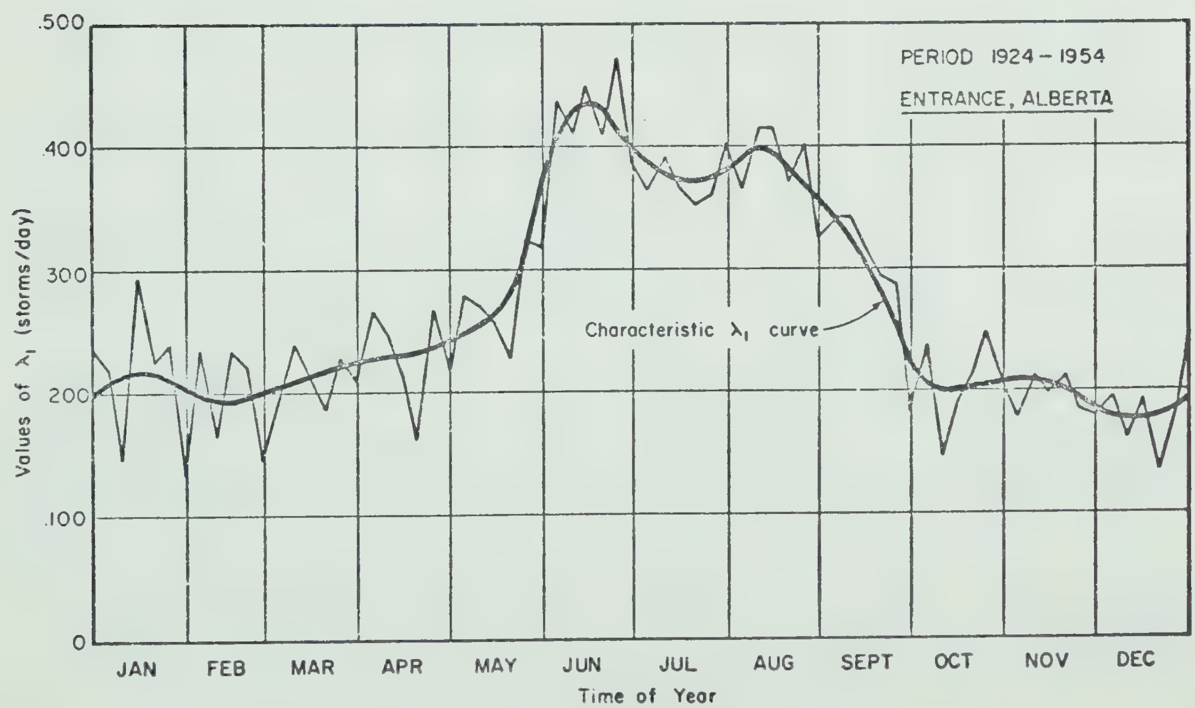

Fig. 15. λ_1 values computed using distribution of n_t

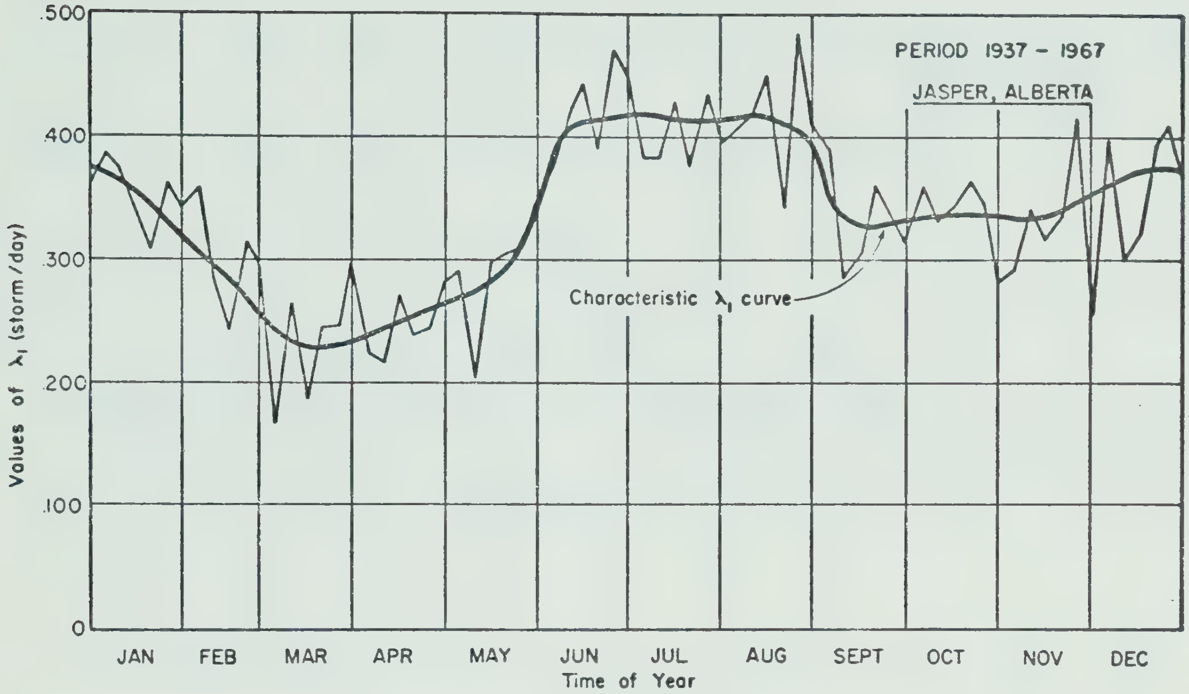

Fig.16. λ_1 values computed using distribution of n_t

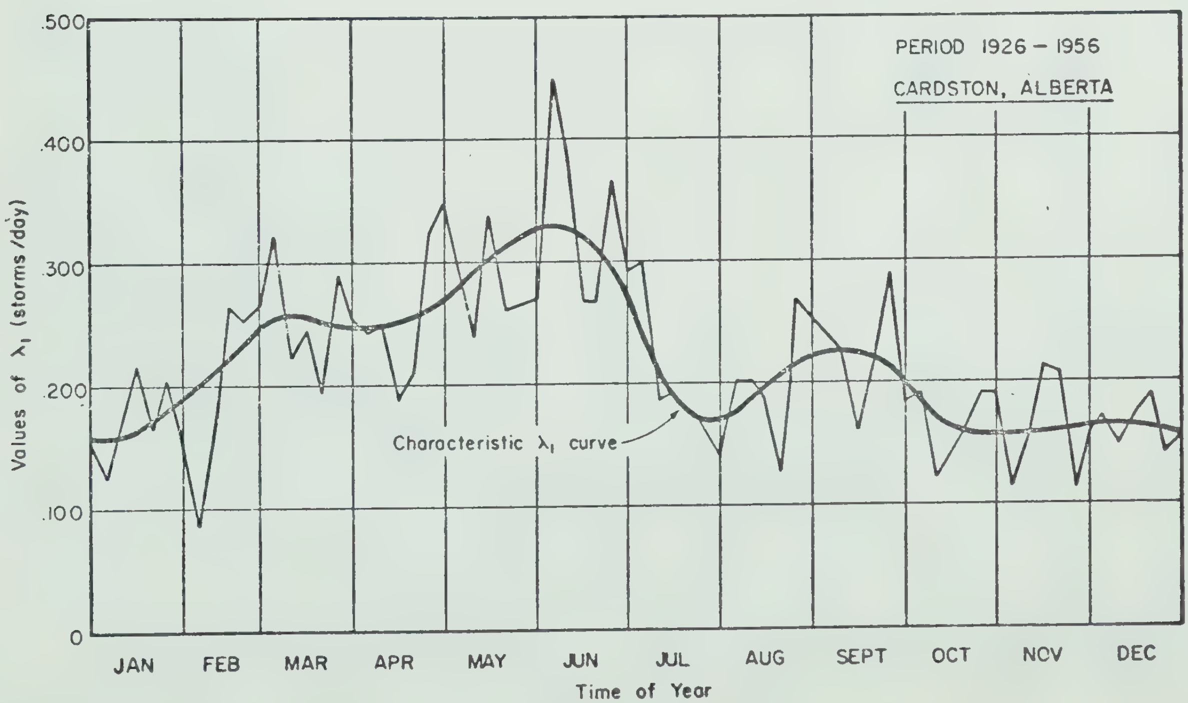

Fig.17. λ_1 values computed using distributed of n_t

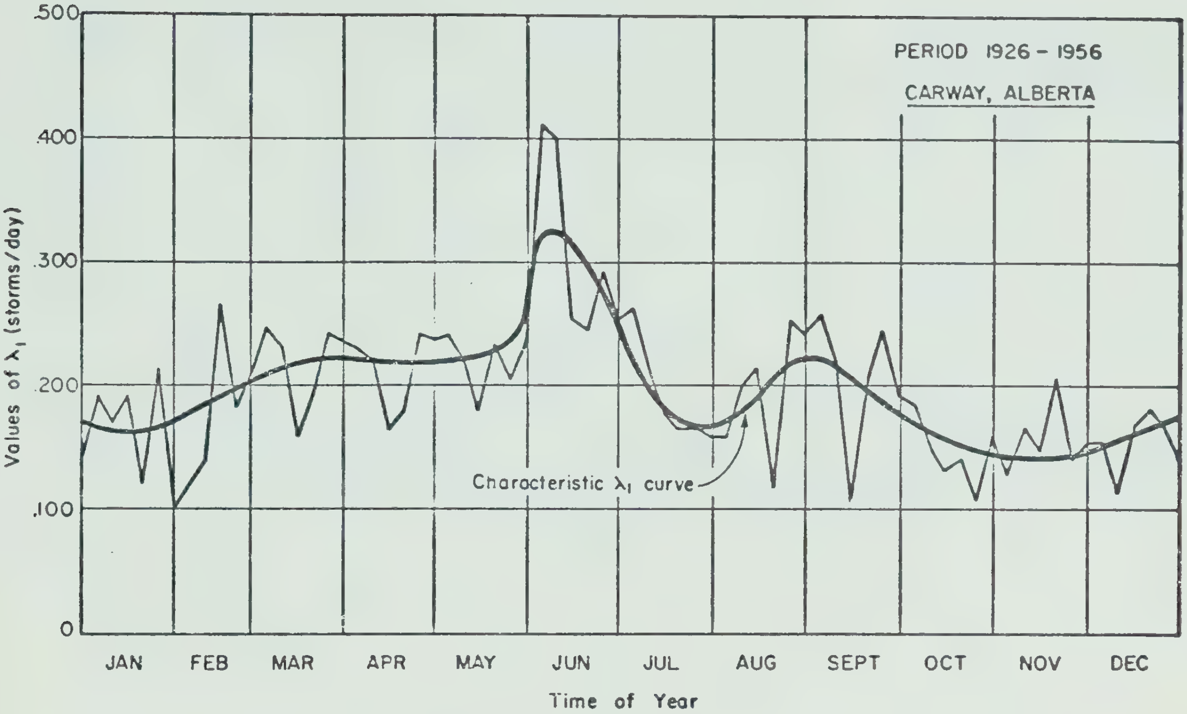

Fig. 18. λ_1 values computed using distribution of n_t

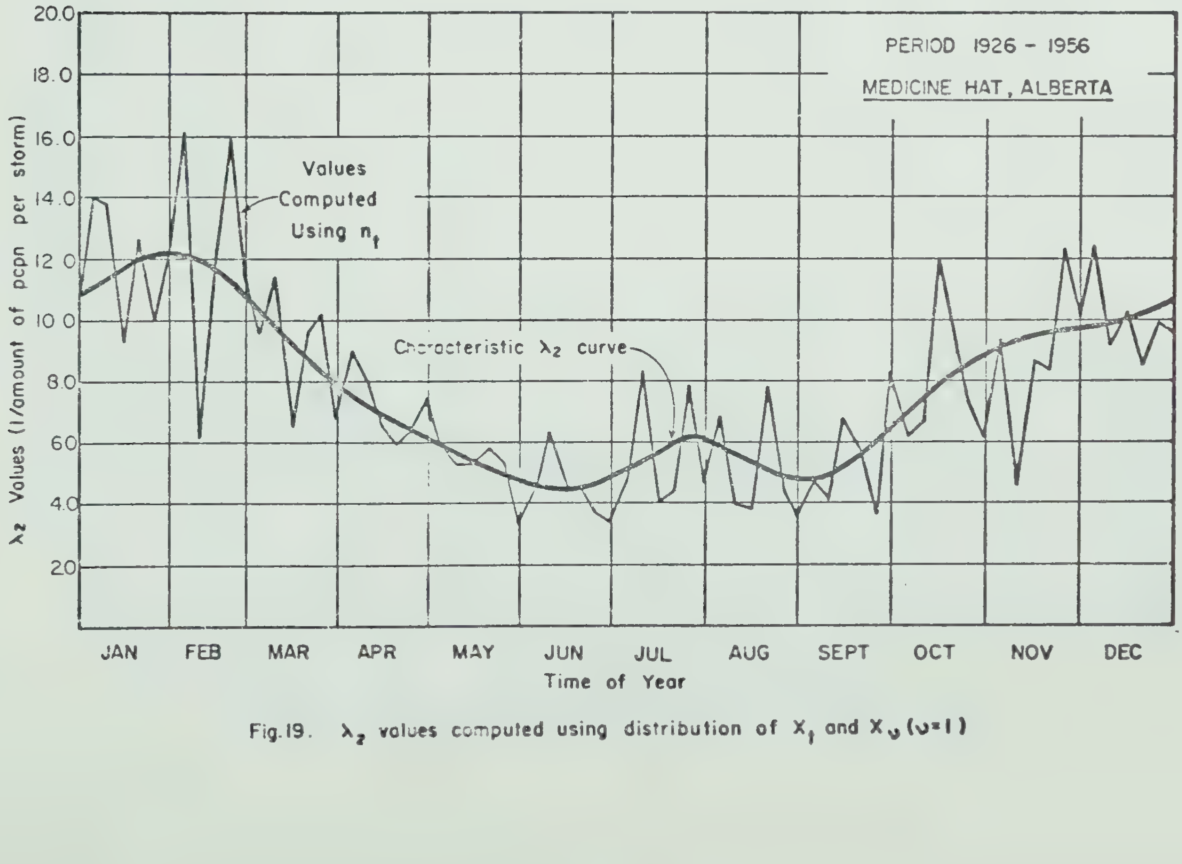

Fig. 19. λ_2 values computed using distribution of X_t and X_υ ($\upsilon = 1$)

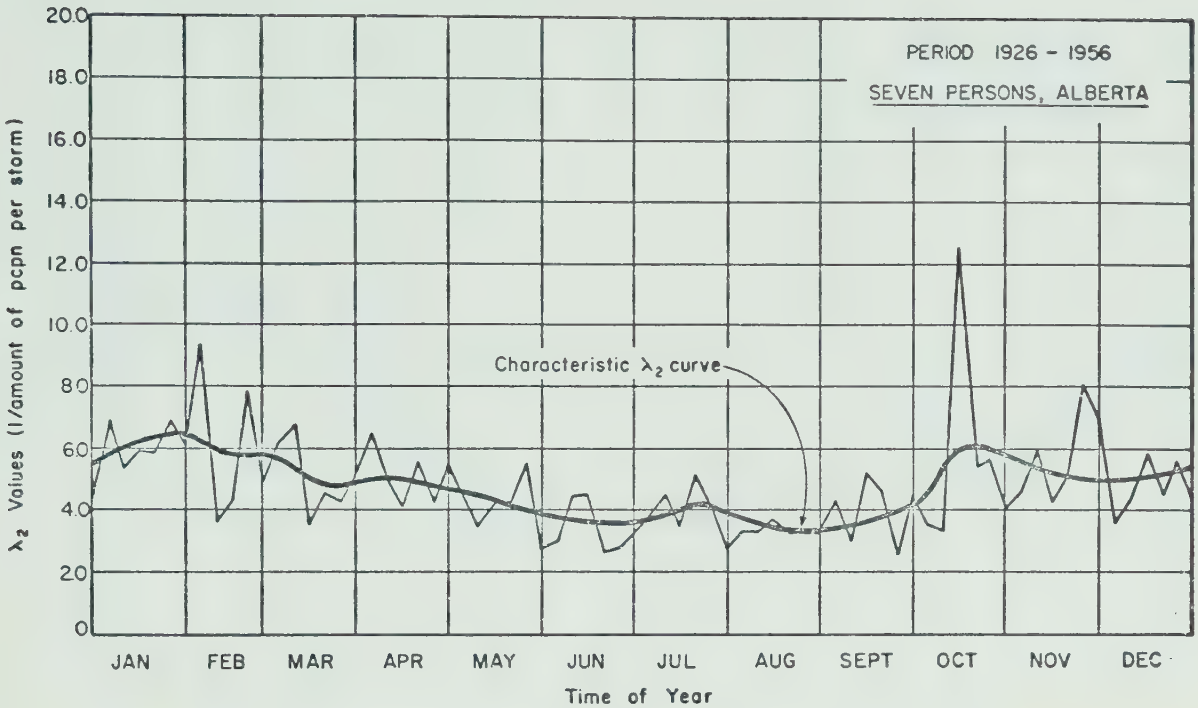

Fig. 20. λ_2 values computed using distribution of X_t and $X_\nu(\nu=1)$

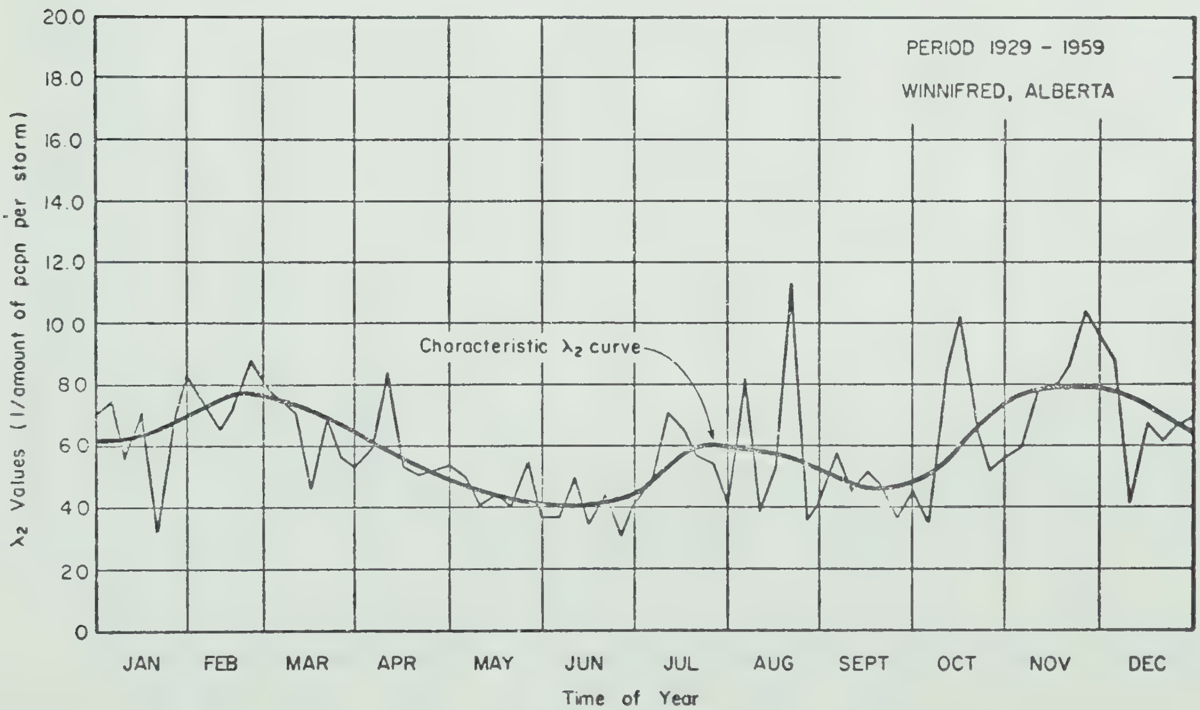

Fig. 21. λ_2 values computed using distribution of X_t and $X_\nu(\nu=1)$

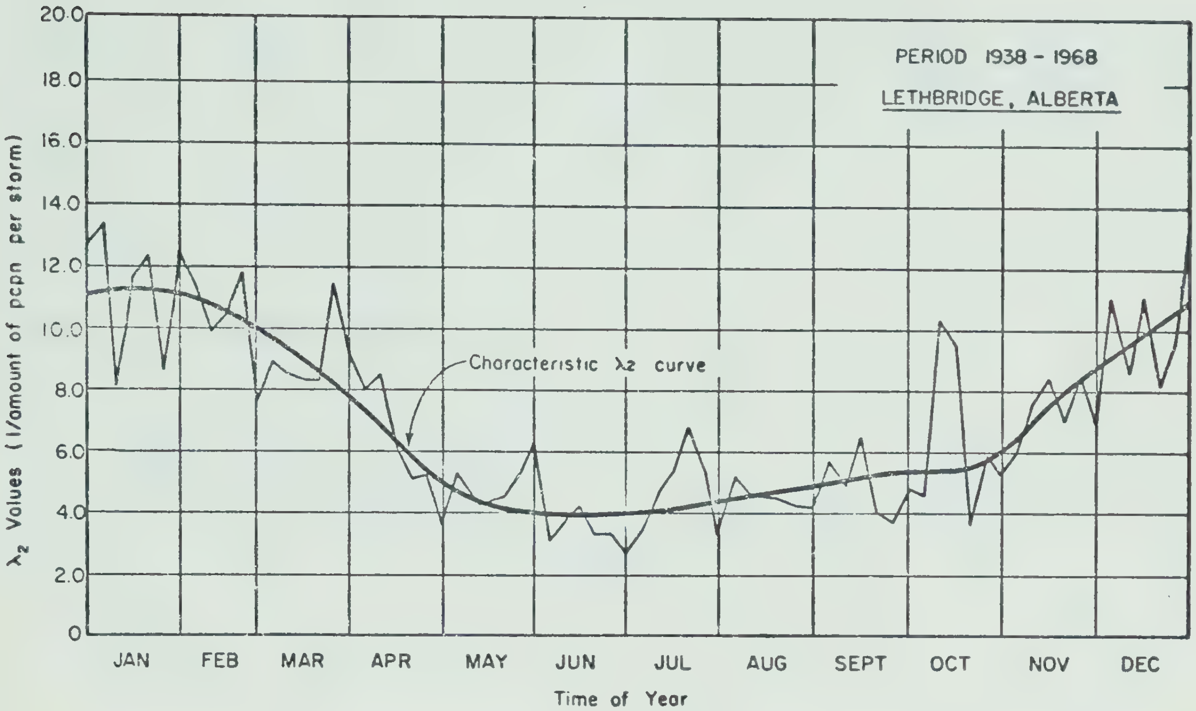

Fig. 22. λ_2 values computed using distribution of X_1 and $X_\nu(\nu=1)$

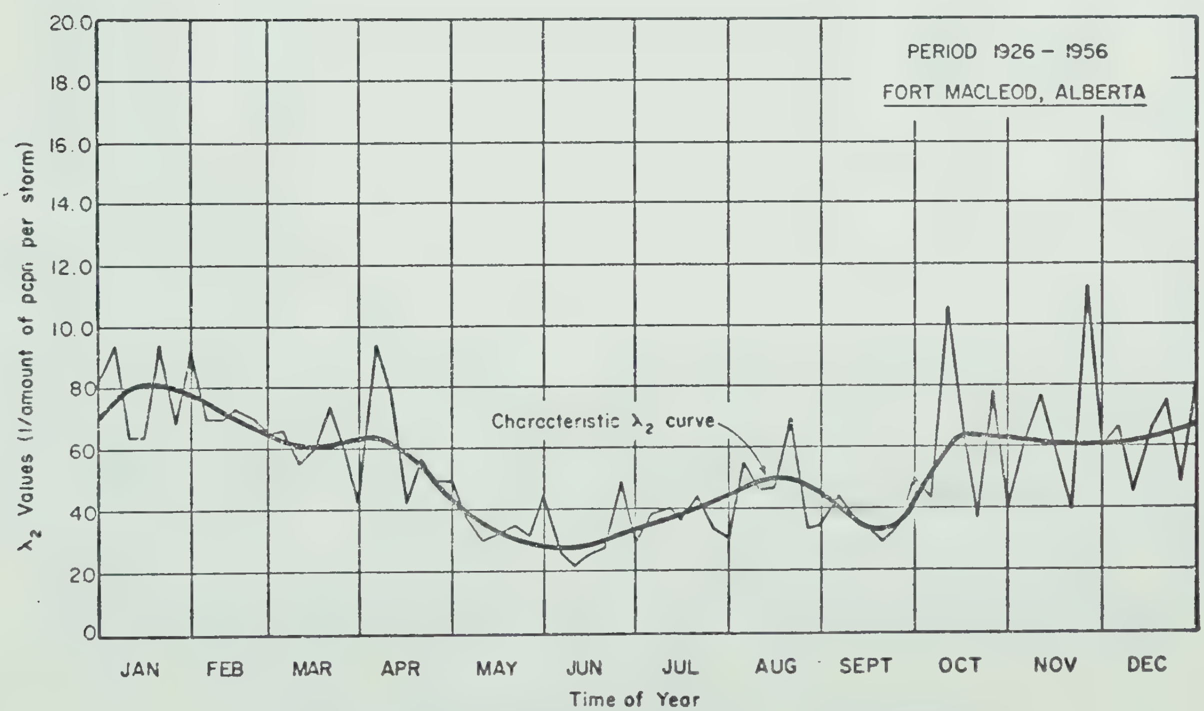

Fig. 23. λ_2 values computed using distribution of X_1 and $X_\nu(\nu=1)$

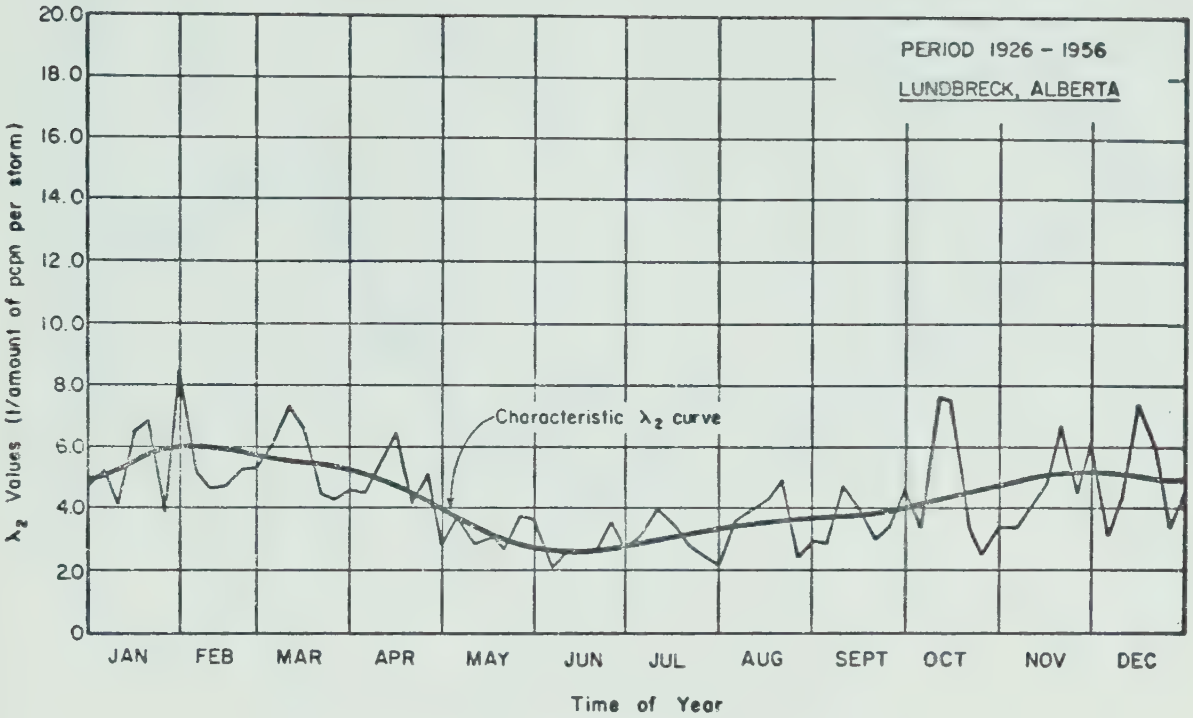

Fig. 24. λ_2 values computed using distribution of X_t and $X_\nu(\nu=1)$

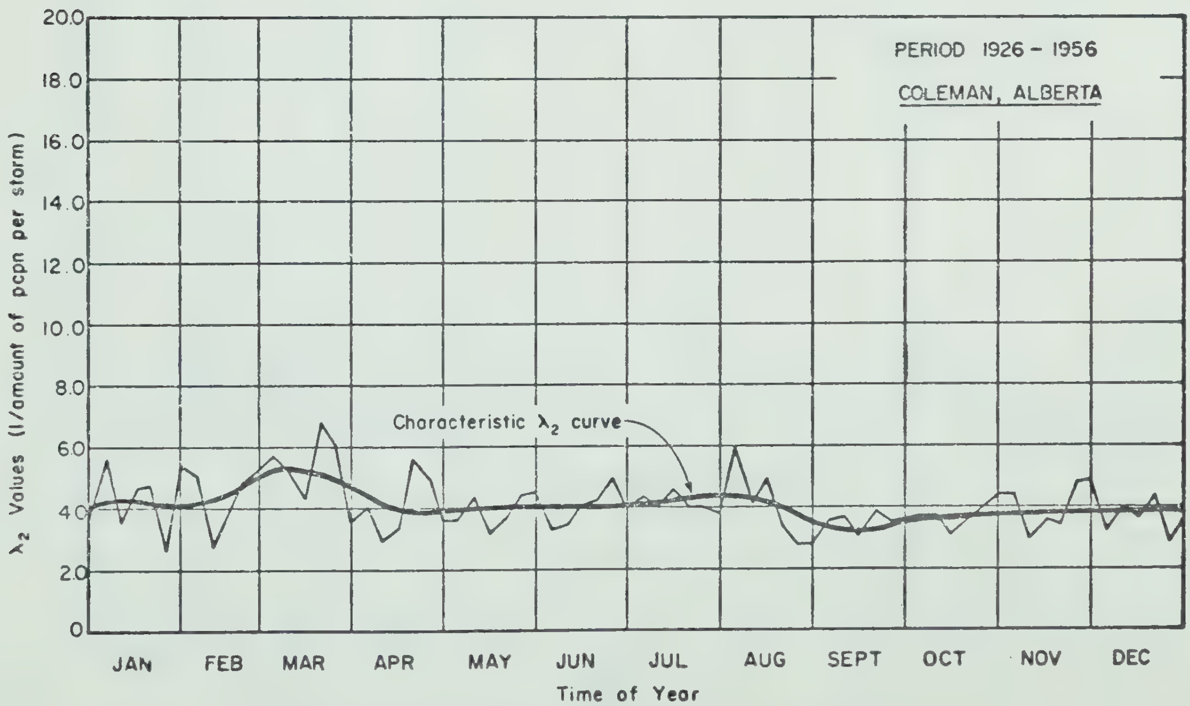

Fig. 25. λ_2 values computed using distribution of X_t and $X_\nu(\nu=1)$

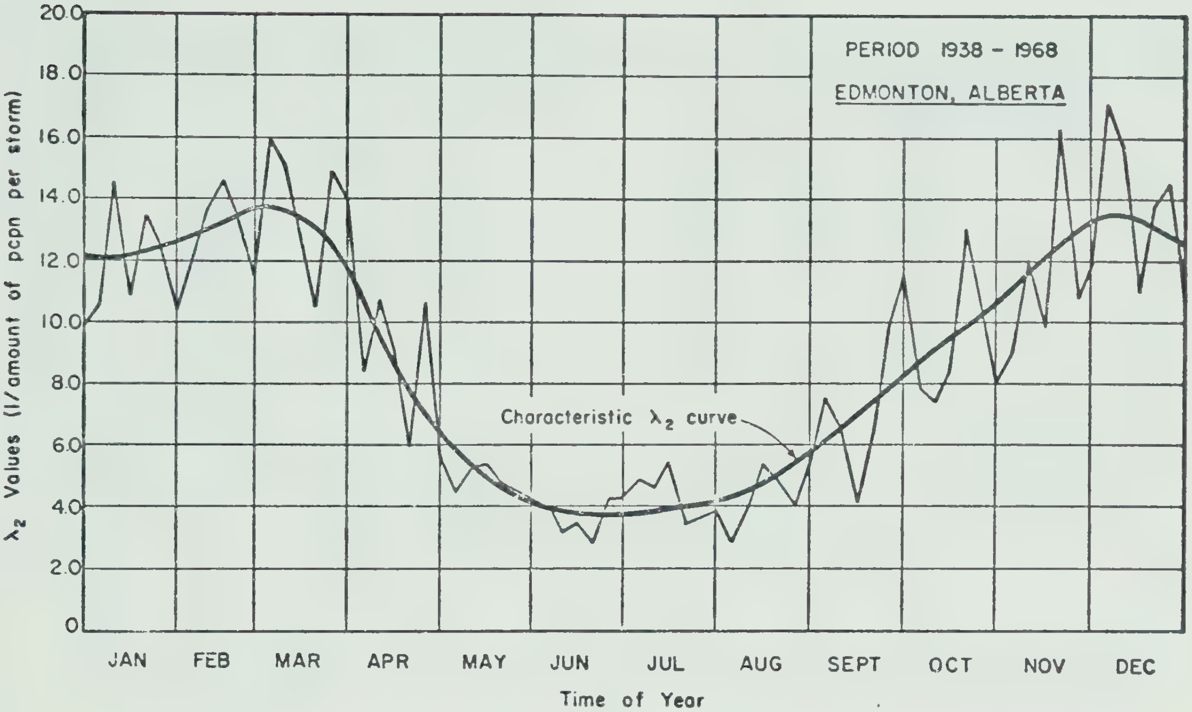

Fig. 26. λ_2 values computed using distribution of X_t and $X_v (v = 1)$

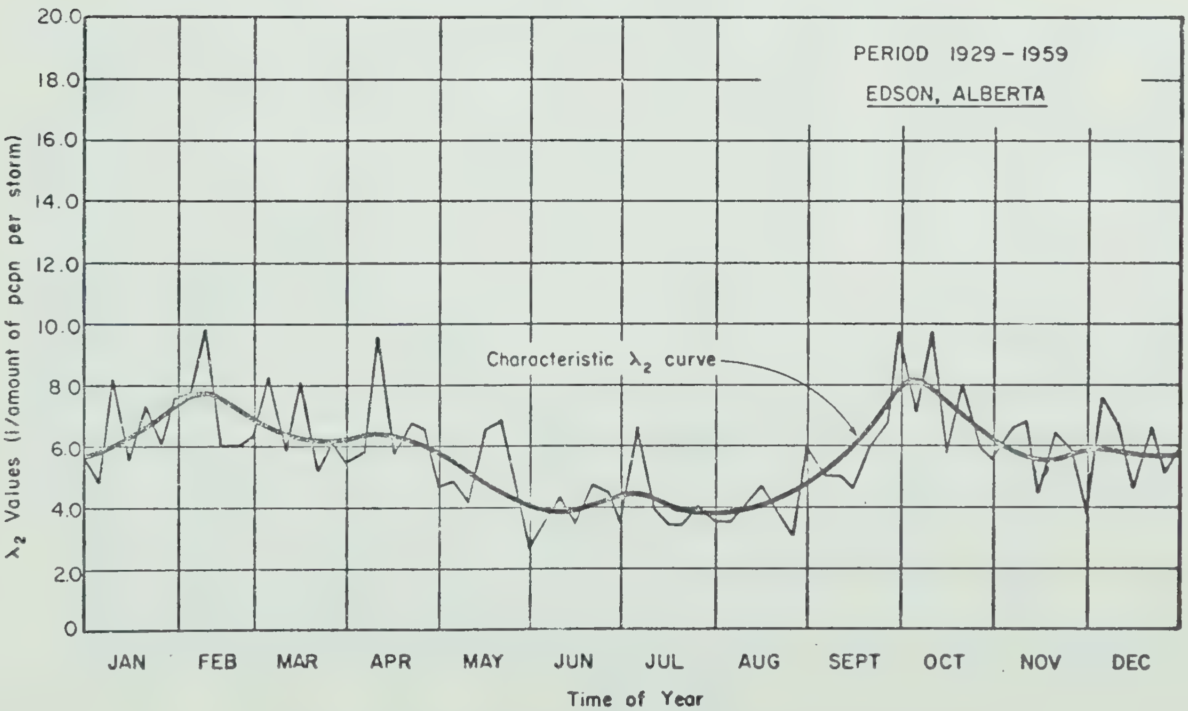

Fig. 27. λ_2 values computed using distribution of X_t and $X_v (v = 1)$

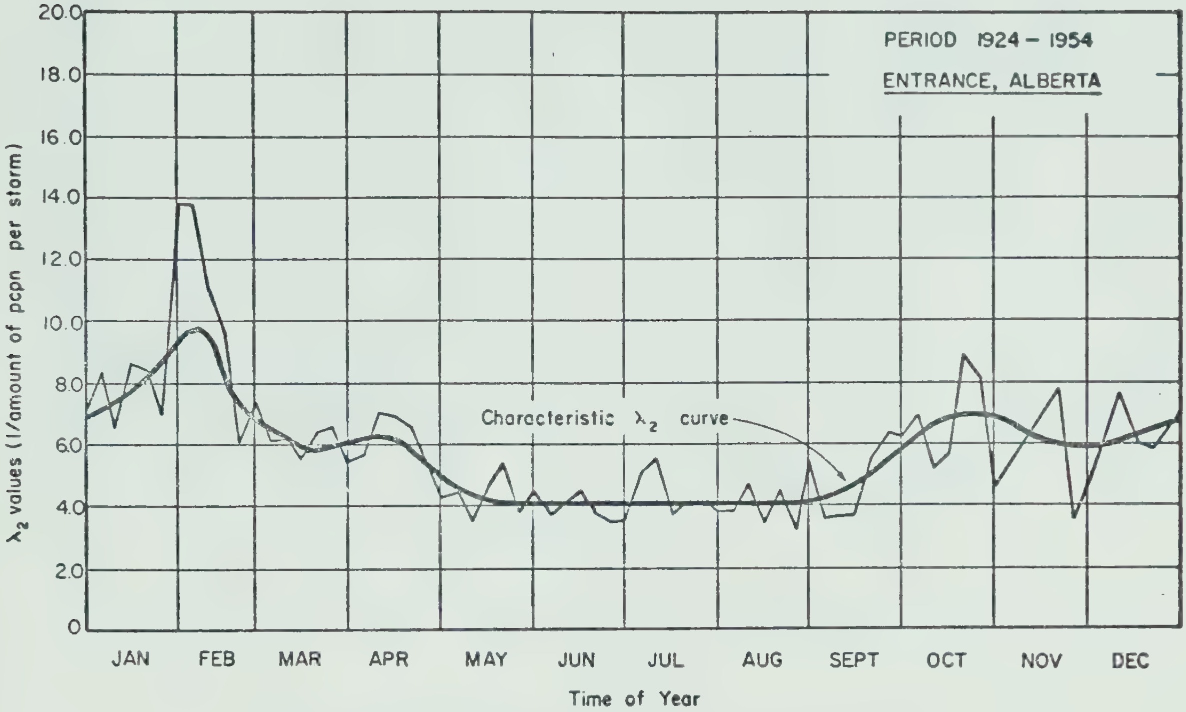

Fig. 28. λ_2 values computed using distribution of X_t and $X_\nu (\nu = 1)$

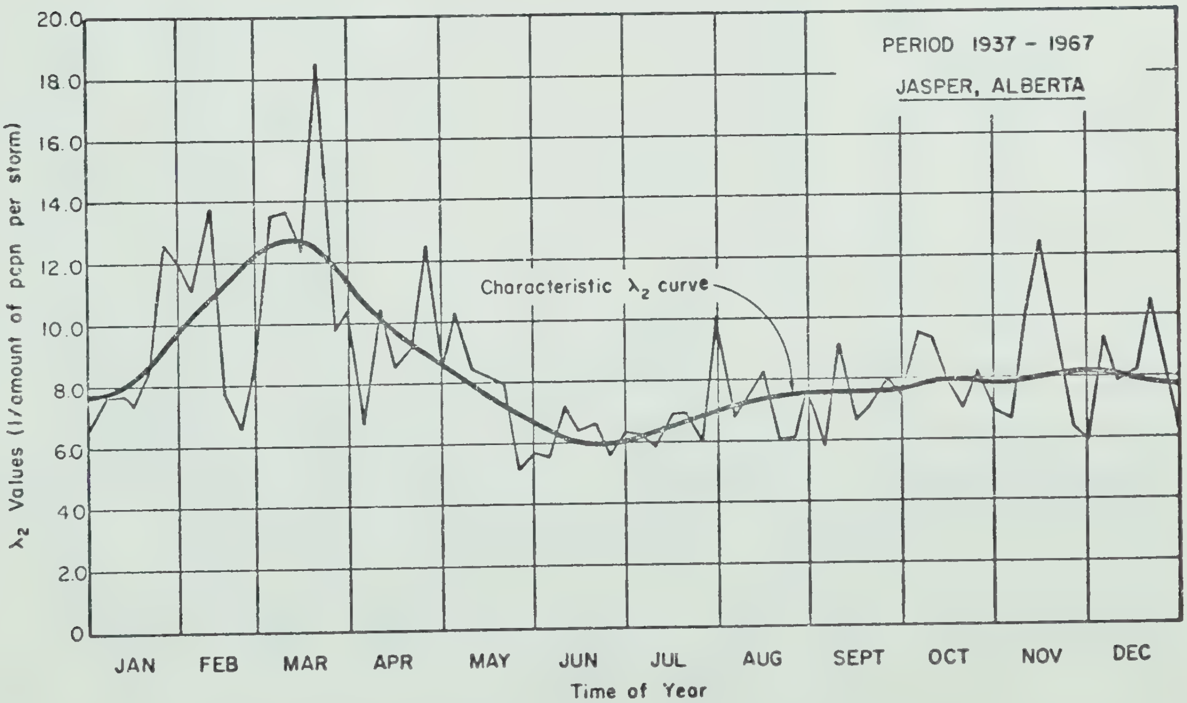

Fig. 29. λ_2 values computed using distribution of X_t and $X_\nu (\nu = 1)$

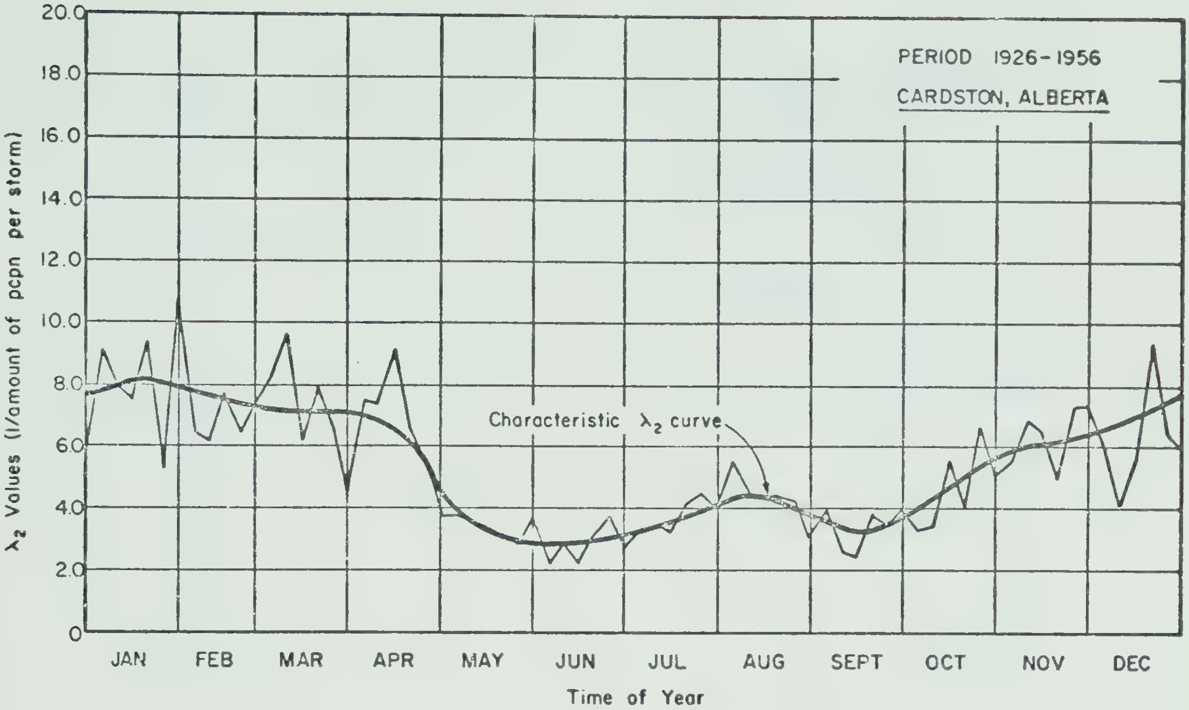

Fig. 30. λ_2 values computed using distribution of X_1 and X_ν (ν=1)

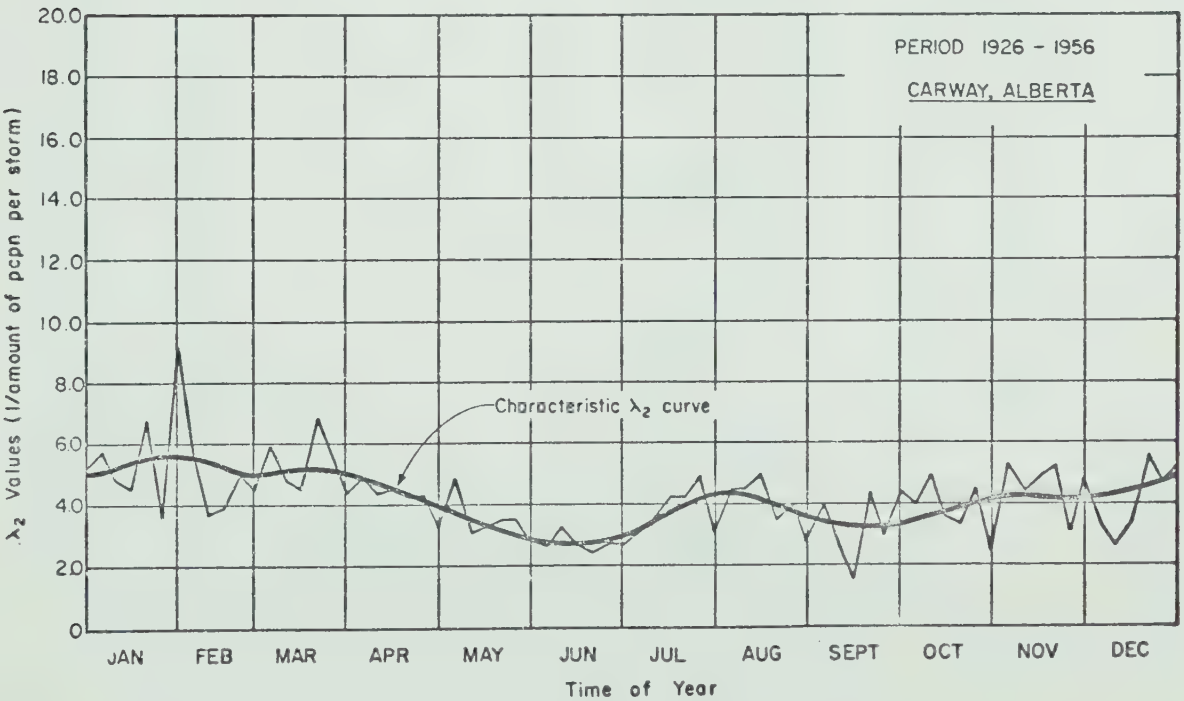

Fig. 31. λ_2 values computed using distribution of X_1 and X_ν (ν=1)

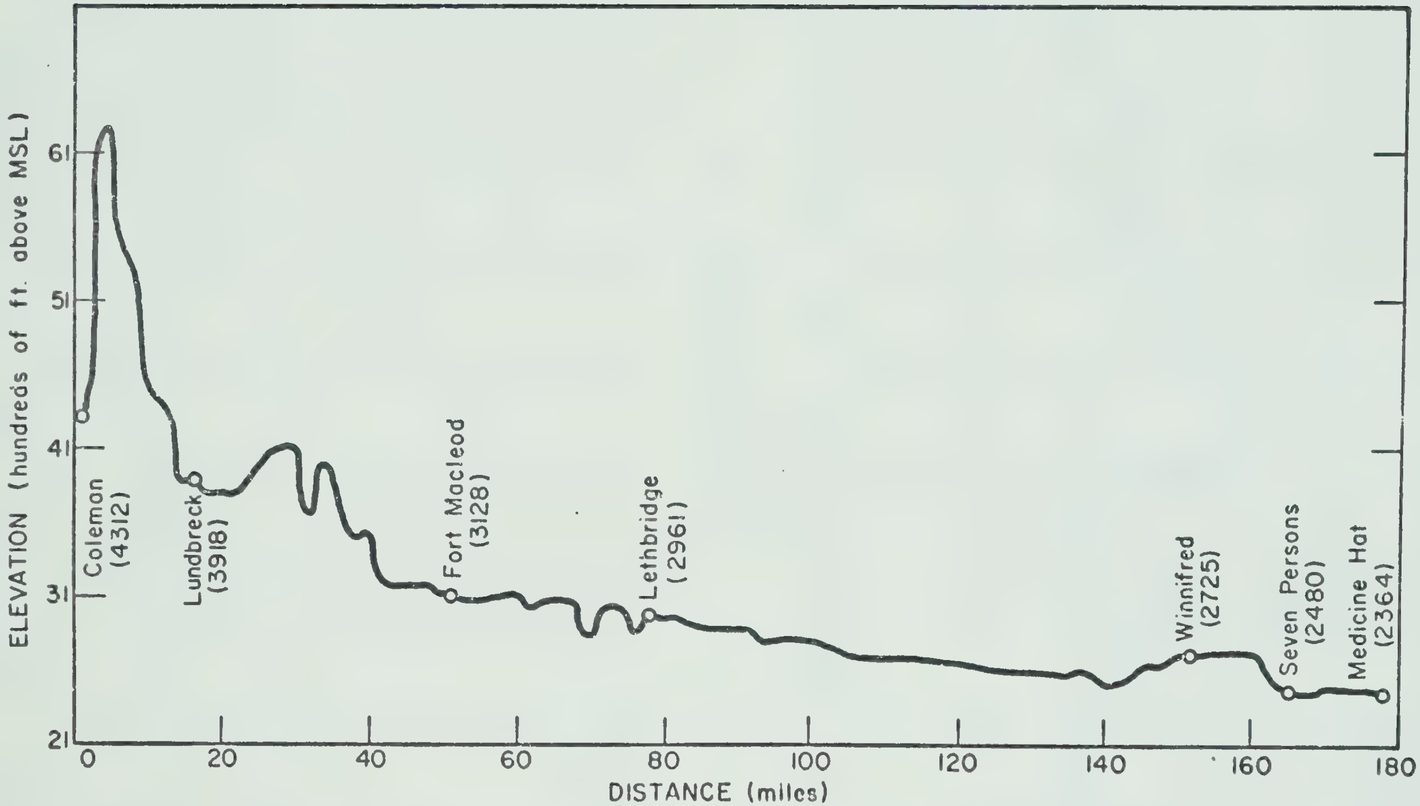

Fig. 32. Elevation profile of some of the southern Alberta stations

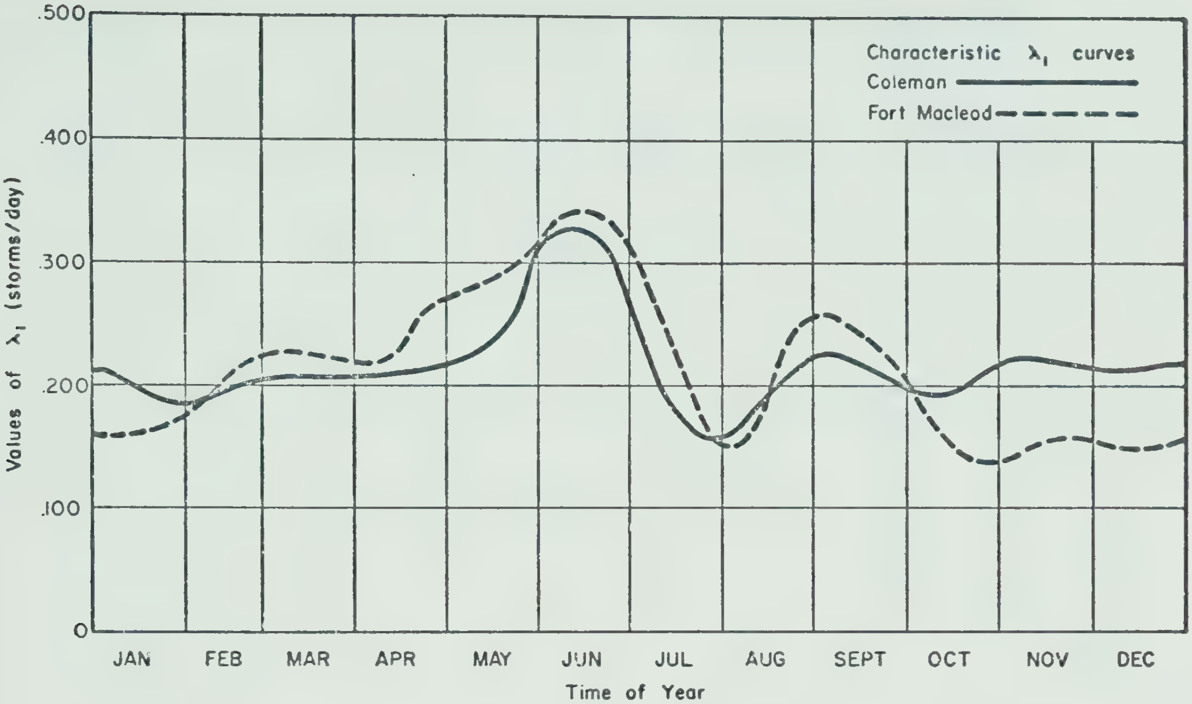

Fig. 33. Comparison of the characteristic λ_1 curves for Coleman and Fort Macleod

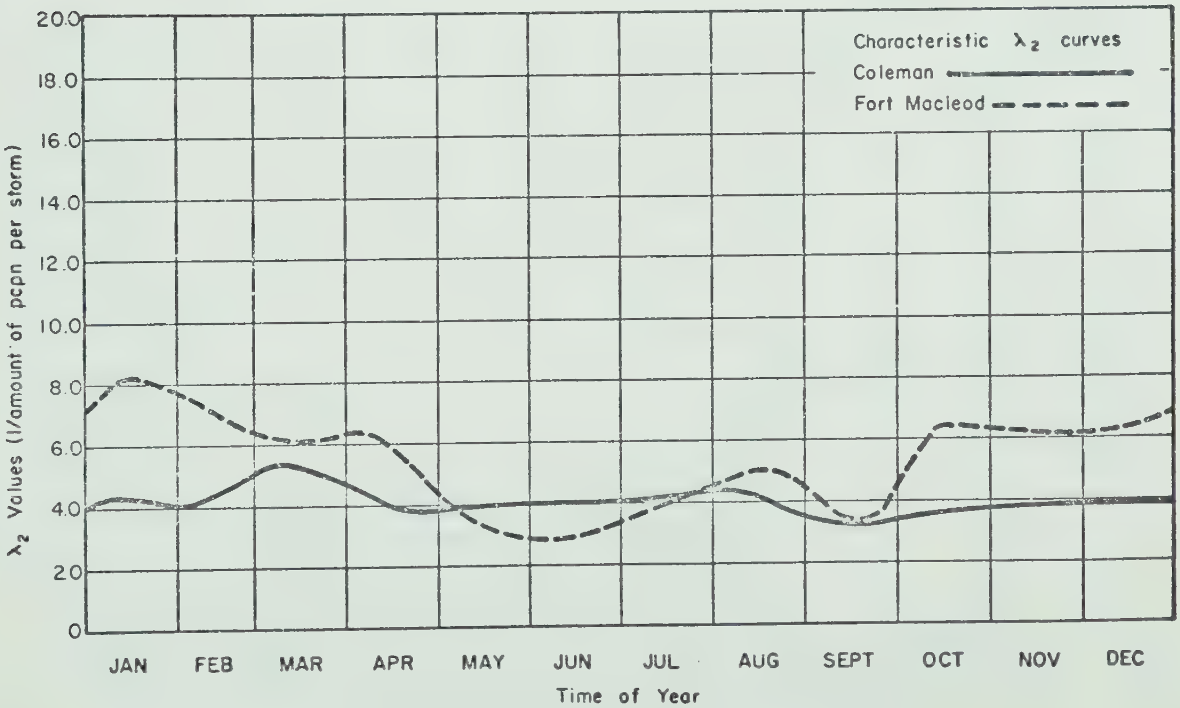

Fig. 34. Comparison of the characteristic λ_2 curves for Coleman and Fort Macleod

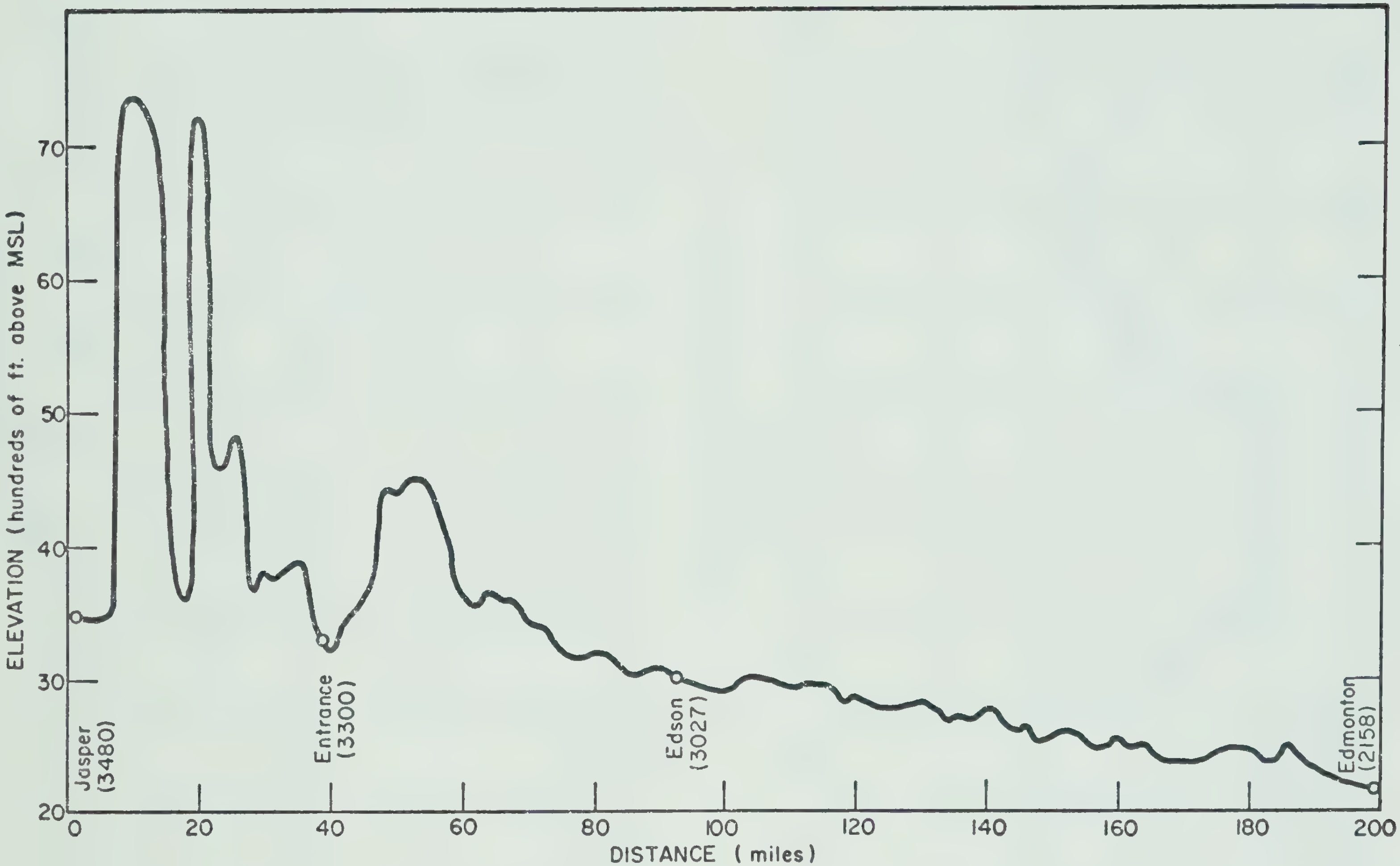

Fig.35. Elevation profile of the central Alberta stations

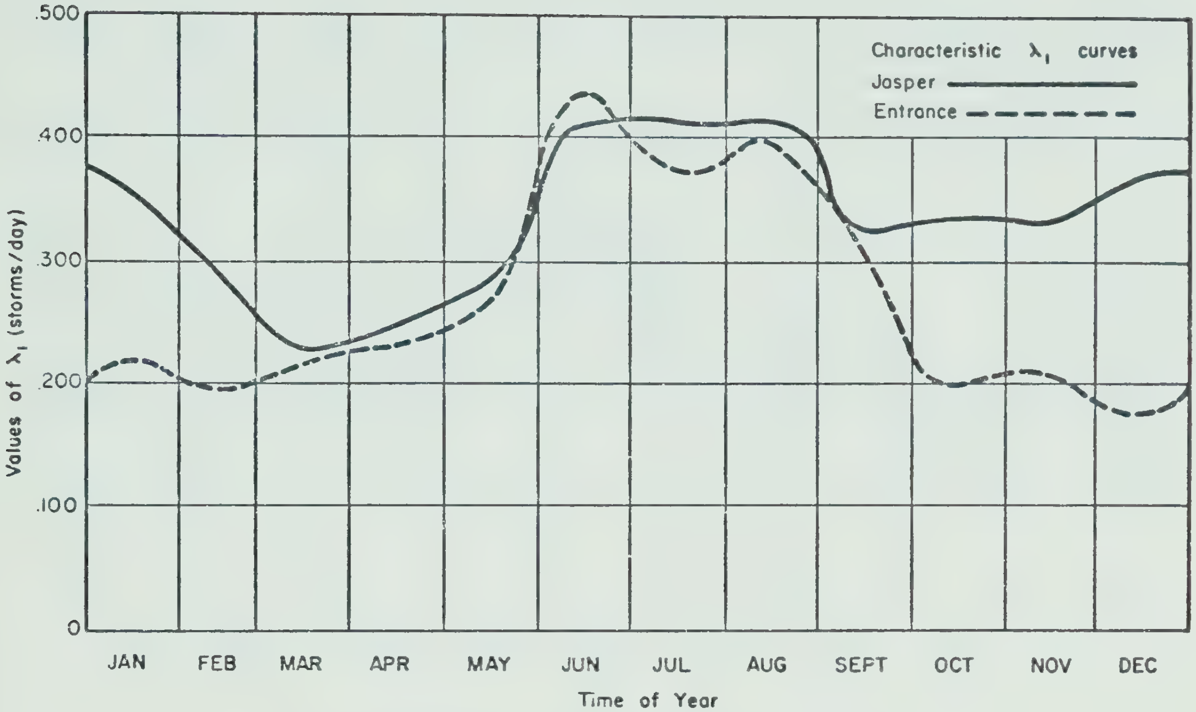

Fig. 36. Comparison of the characteristic λ_1 curves for Jasper and Entrance

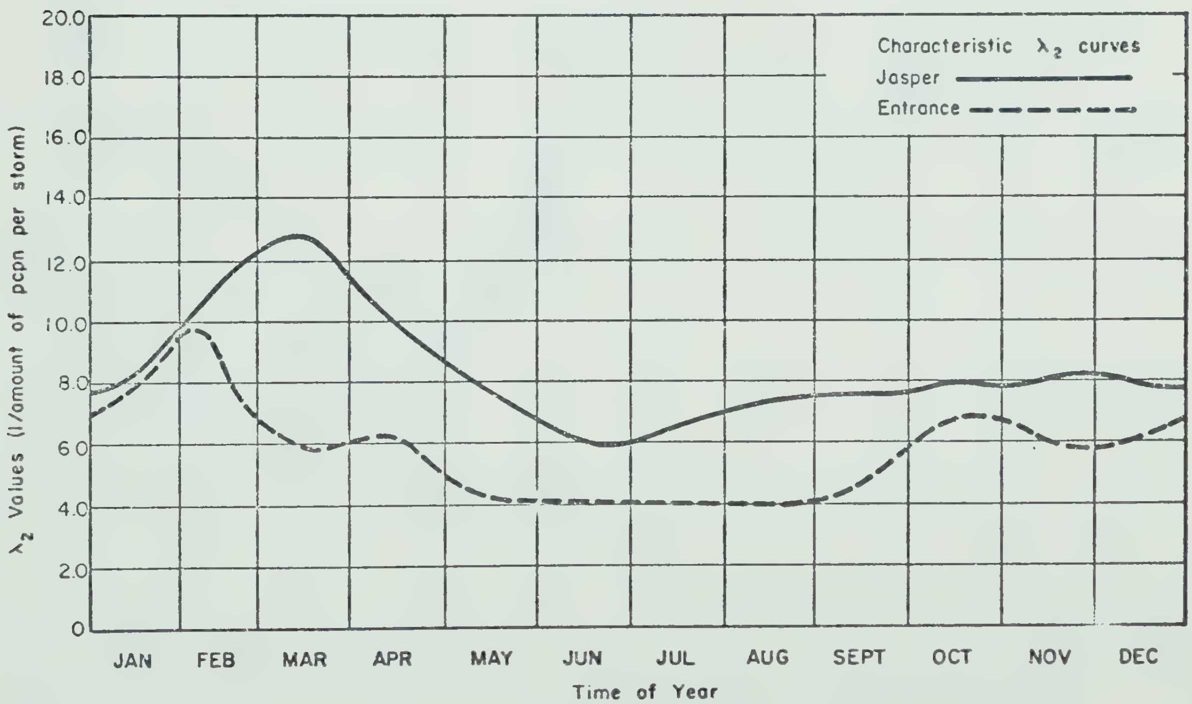

Fig. 37. Comparison of the characteristic λ_2 curves for Jasper and Entrance

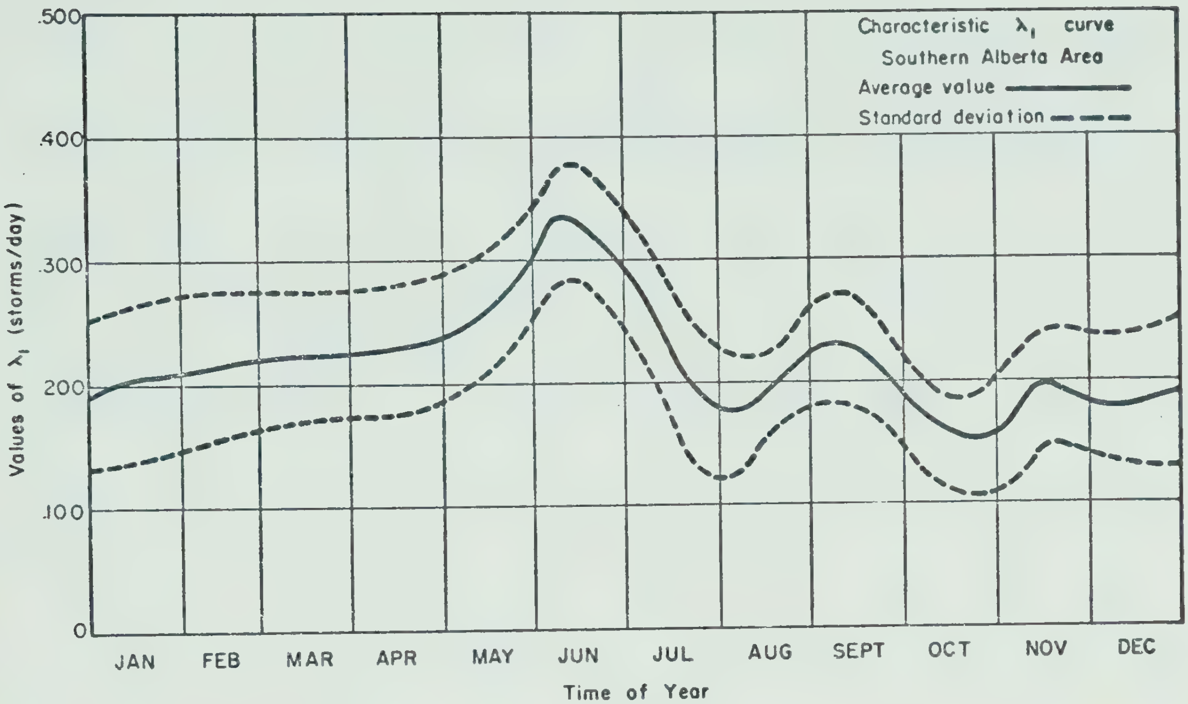

Fig. 38. Characteristic λ_1 curve with first standard deviation for the southern Alberta stations

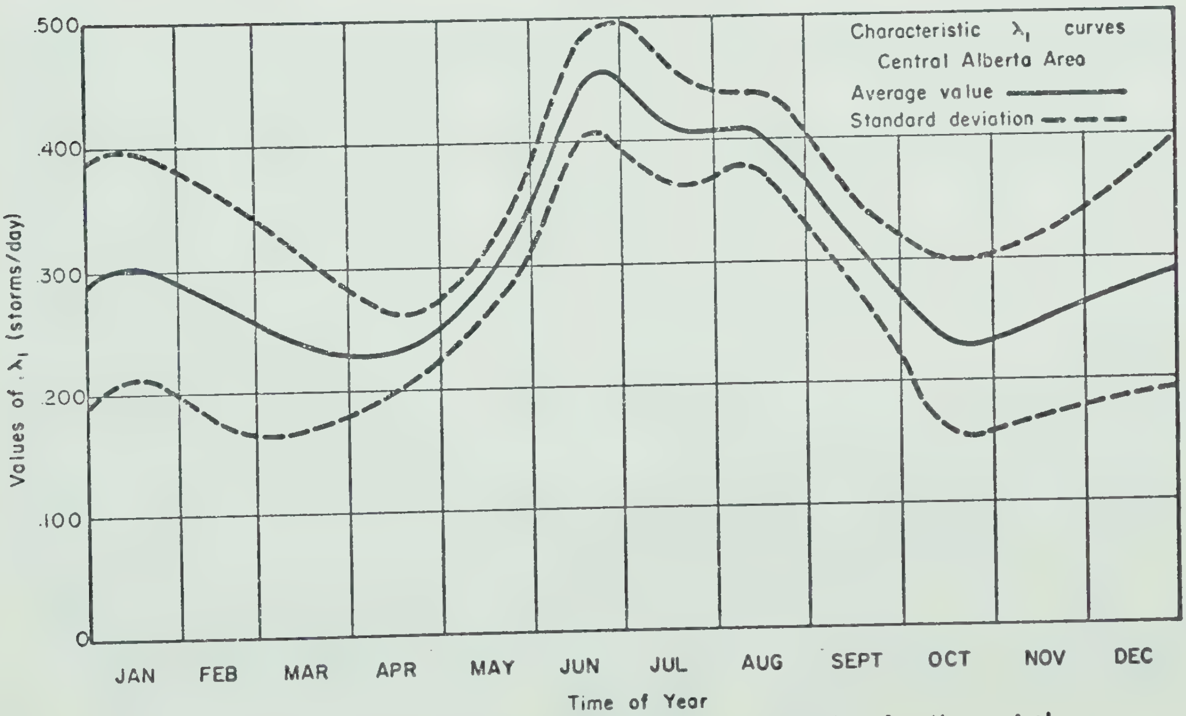

Fig. 39. Characteristic λ_1 curve with first standard deviation for the central Alberta stations

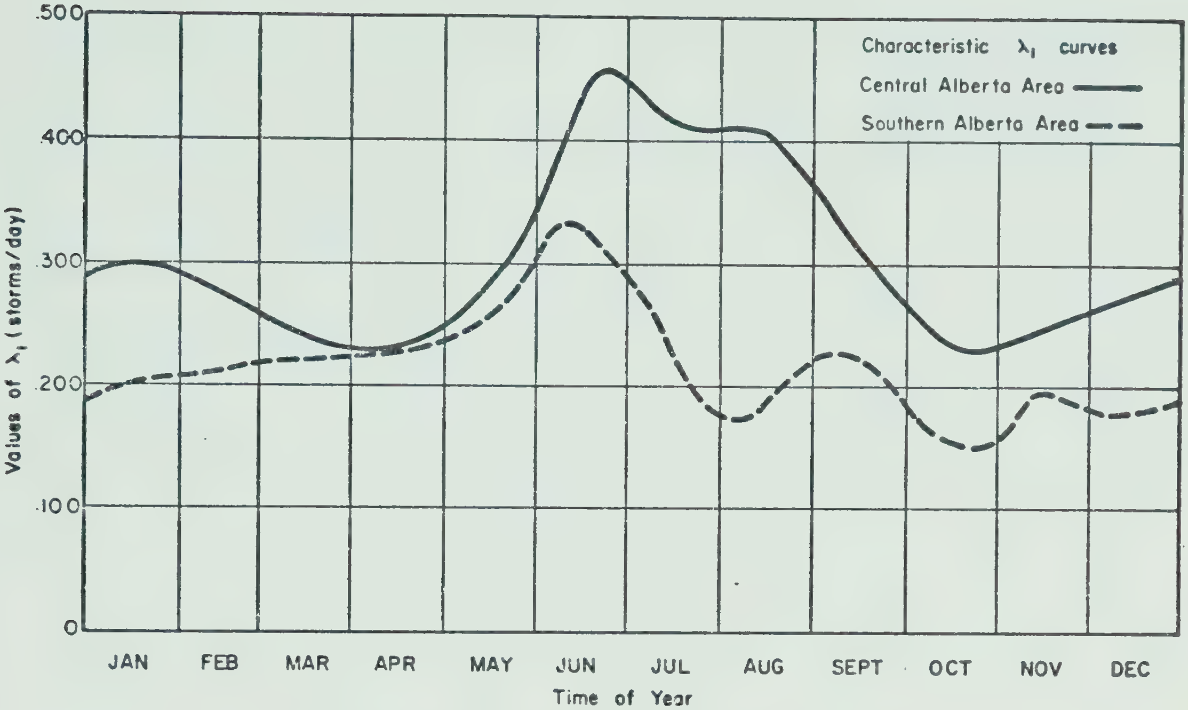

Fig. 40. Comparison of the characteristic λ_1 curves for central and southern Alberta

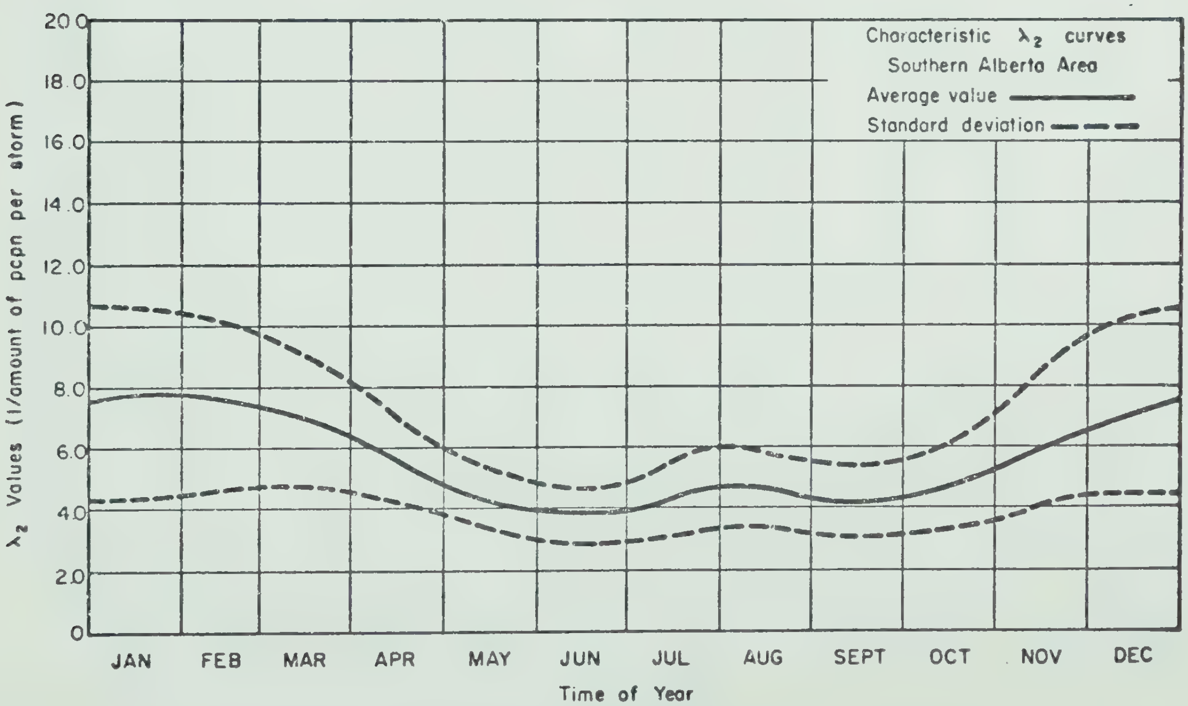

Fig. 41. Characteristic λ_2 curve with first standard deviation for the southern Alberta stations

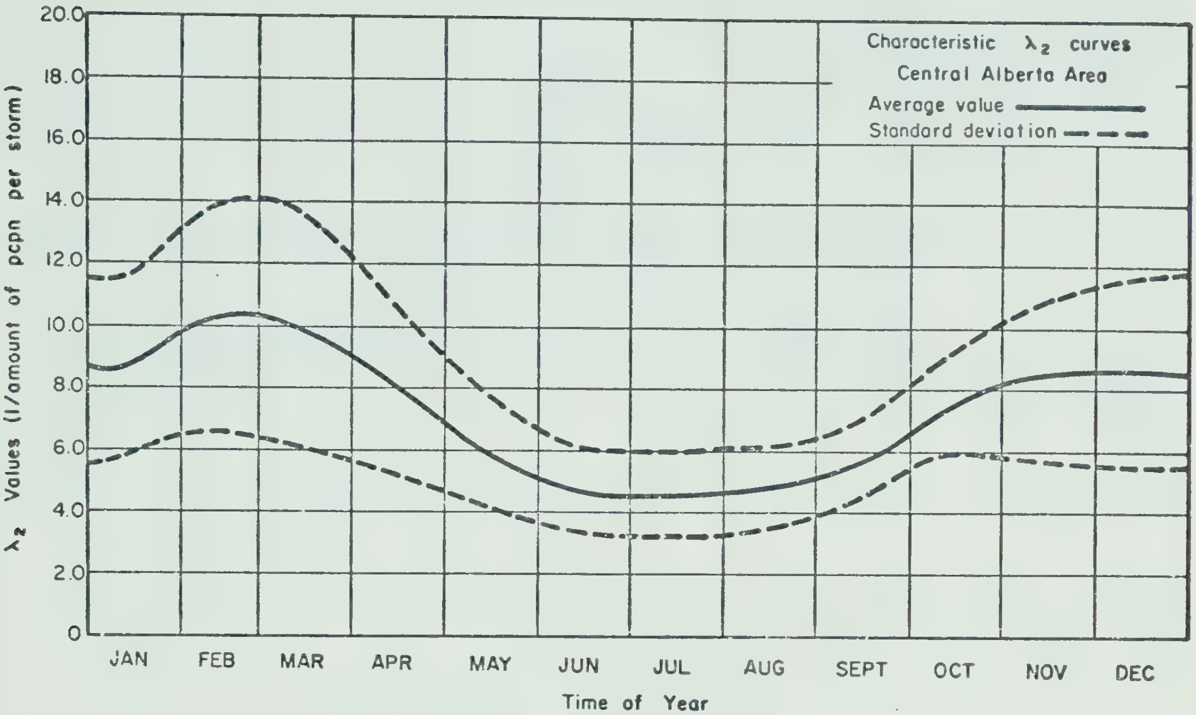

Fig. 42. Characteristic λ_2 curve with first standard deviation for the central Alberta stations

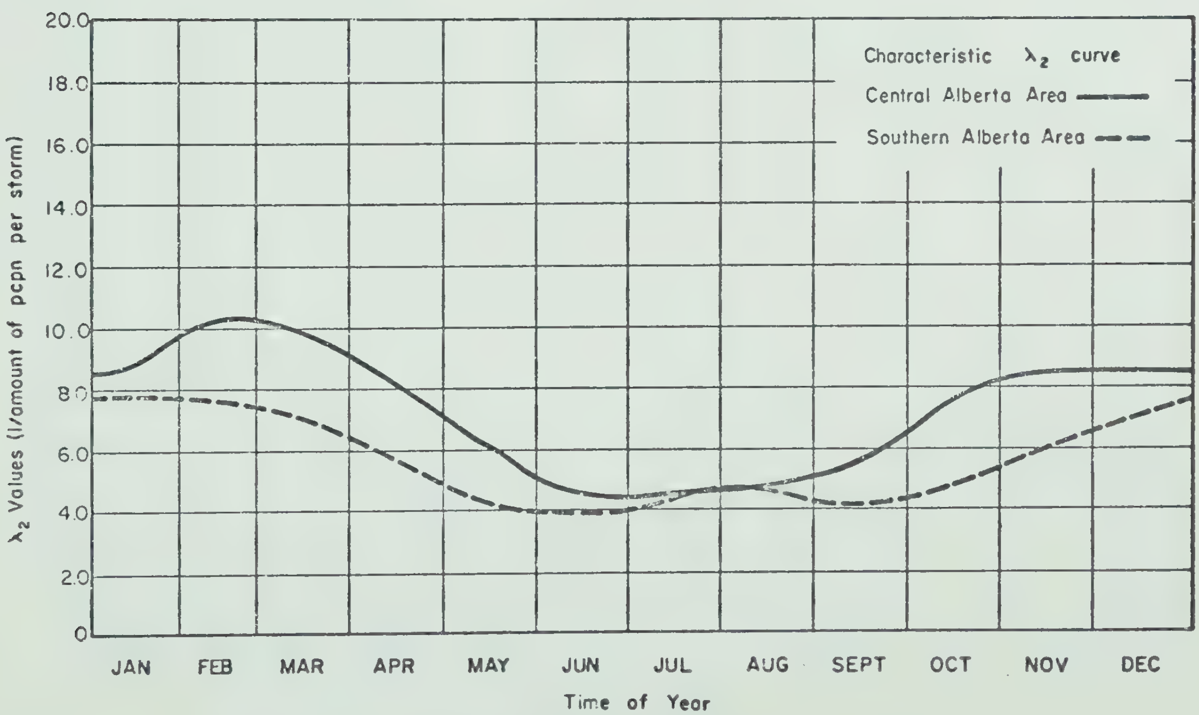

Fig. 43. Comparison of the characteristic λ_2 curves for southern and central Alberta

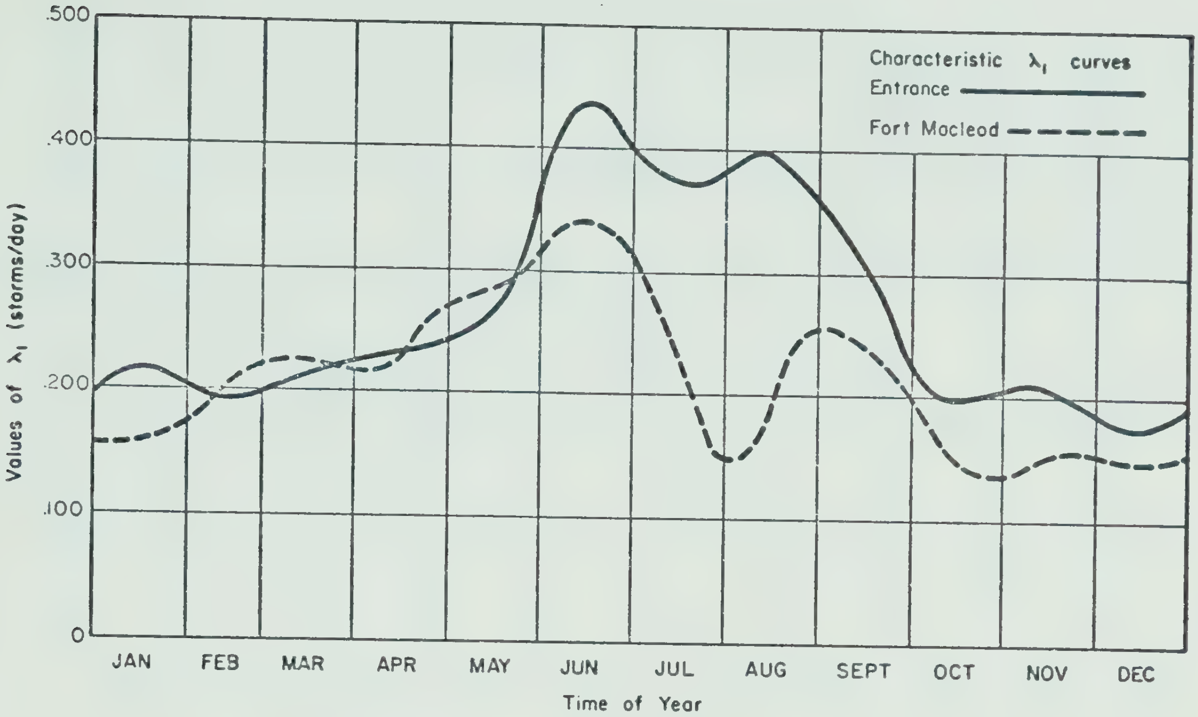

Fig. 44. Comparison of the characteristic λ_1 curves for Fort Macleod and Entrance

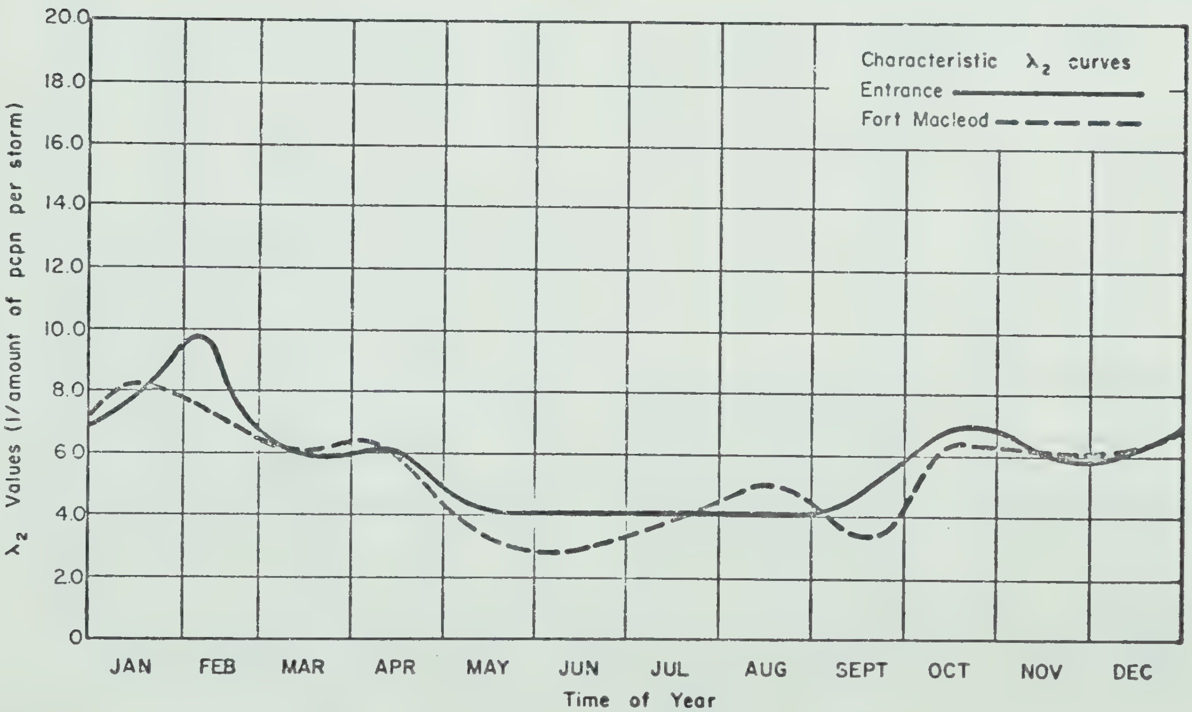

Fig. 45. Comparison of the characteristic λ_2 curves for Fort Macleod and Entrance

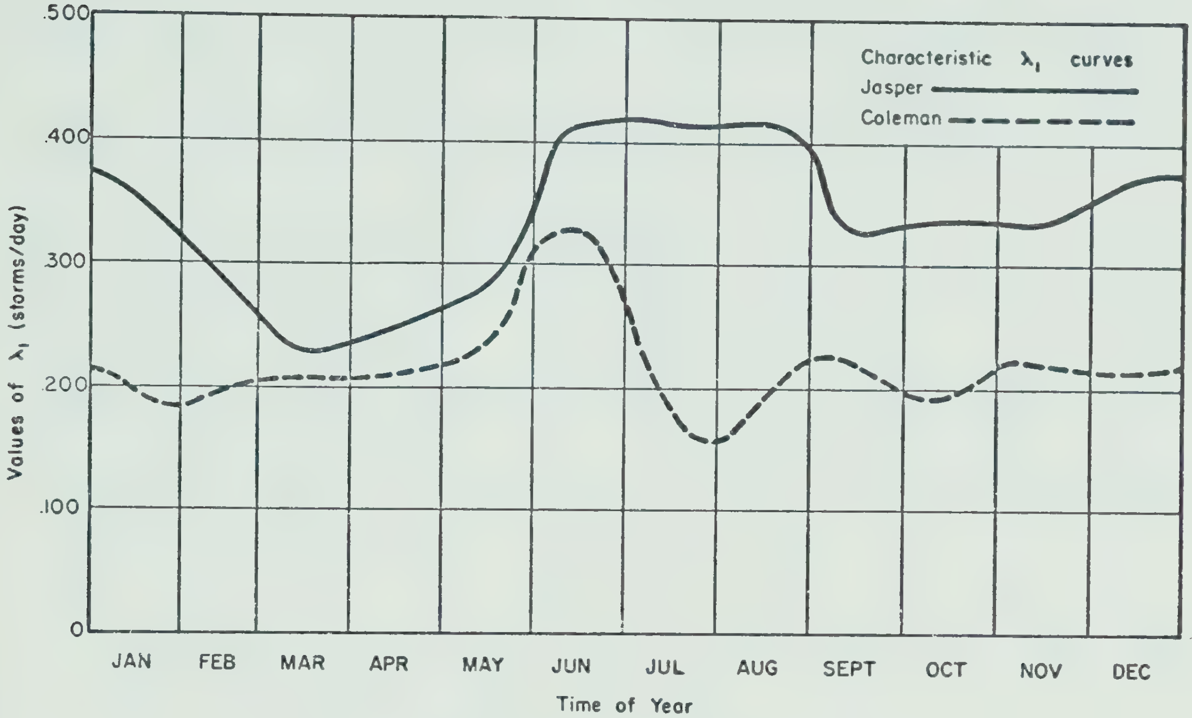

Fig. 46. Comparison of the characteristic λ_1 curves for Jasper and Coleman

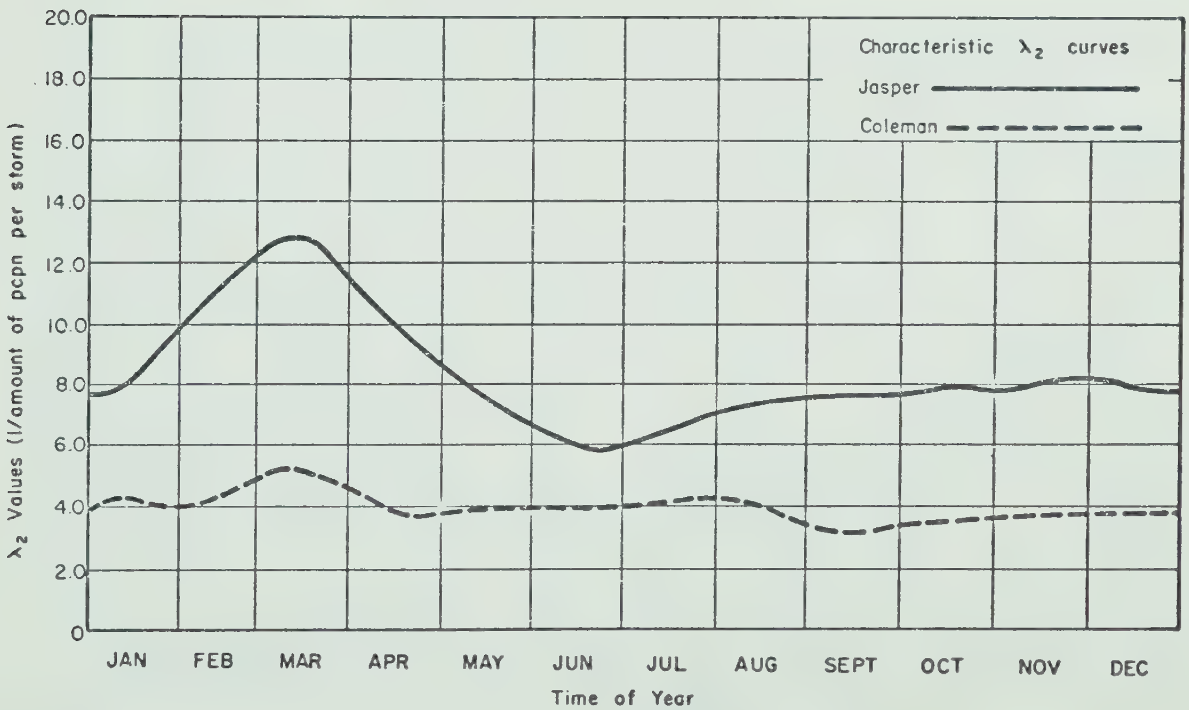

Fig. 47. Comparison of the characteristic λ_2 curves for Jasper and Coleman

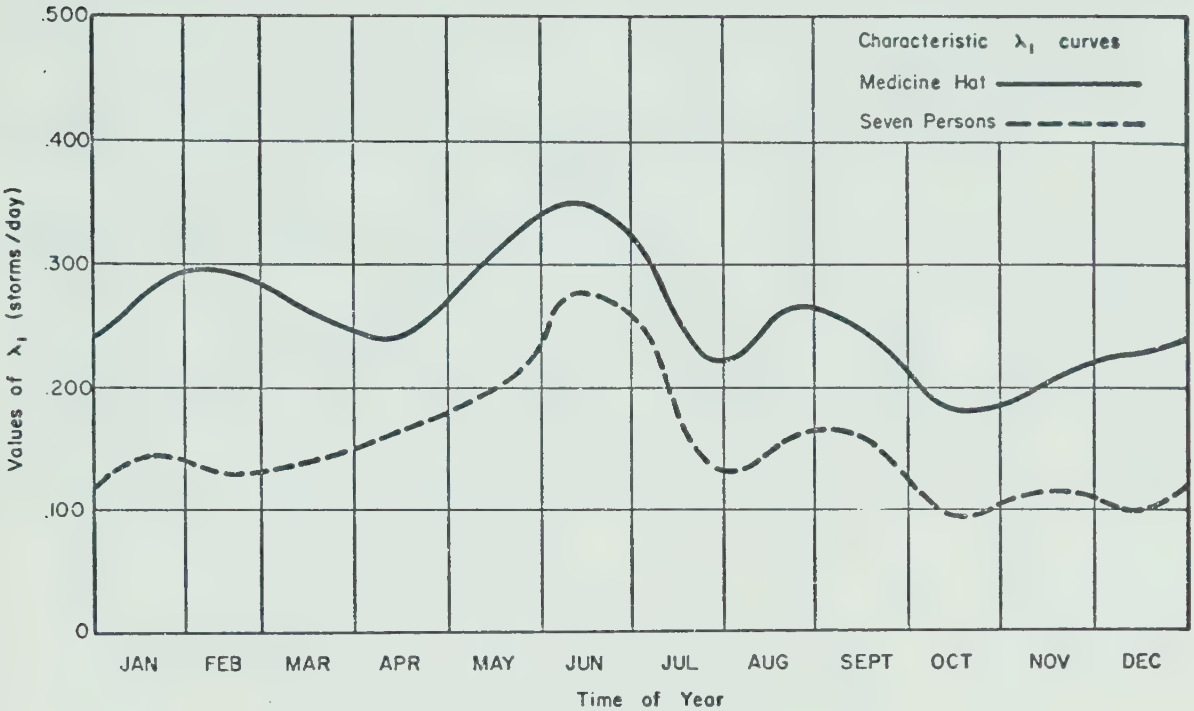

Fig. 48. Comparison of the characteristic λ_1 curves for Medicine Hat and Seven Persons

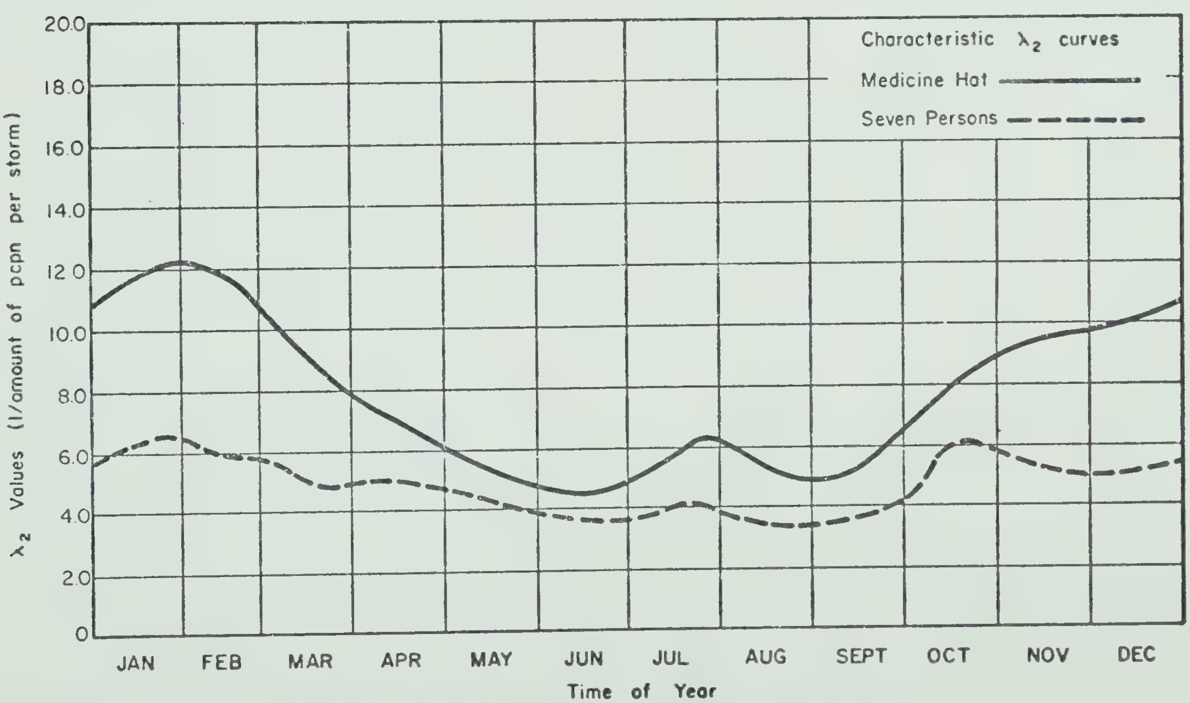

Fig. 49. Comparison of the characteristic λ_2 curves for Medicine Hat and Seven Persons

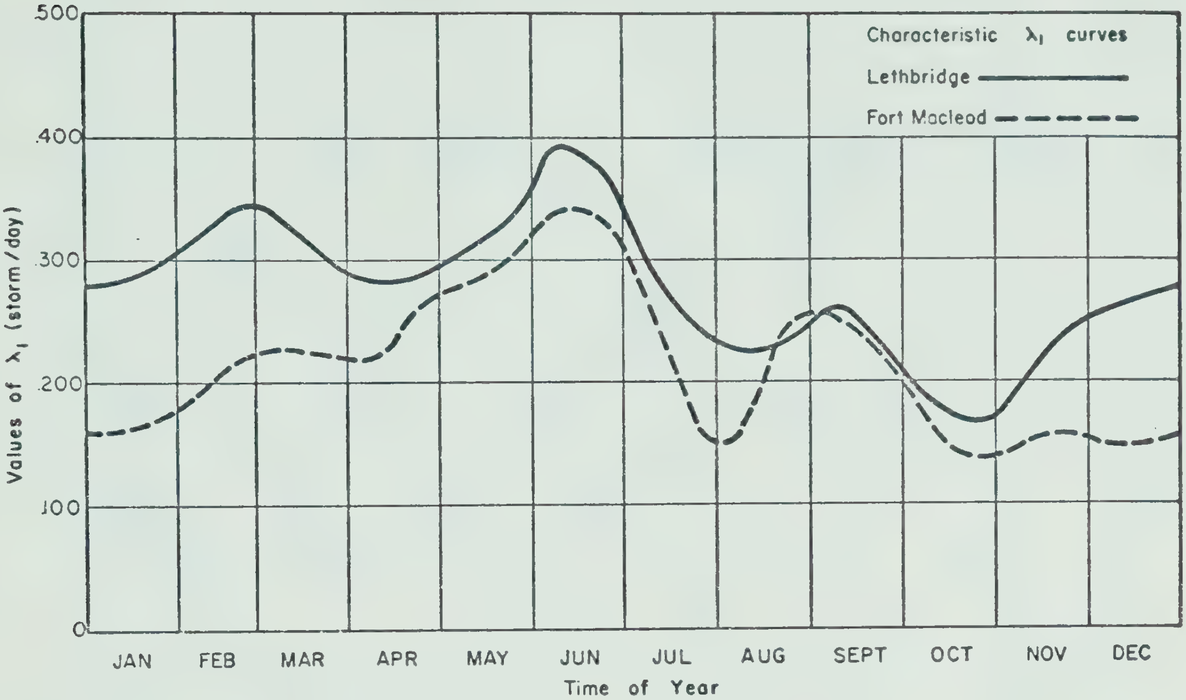

Fig. 50 Comparison of the characteristic λ_1 curves for Lethbridge and Fort Macleod

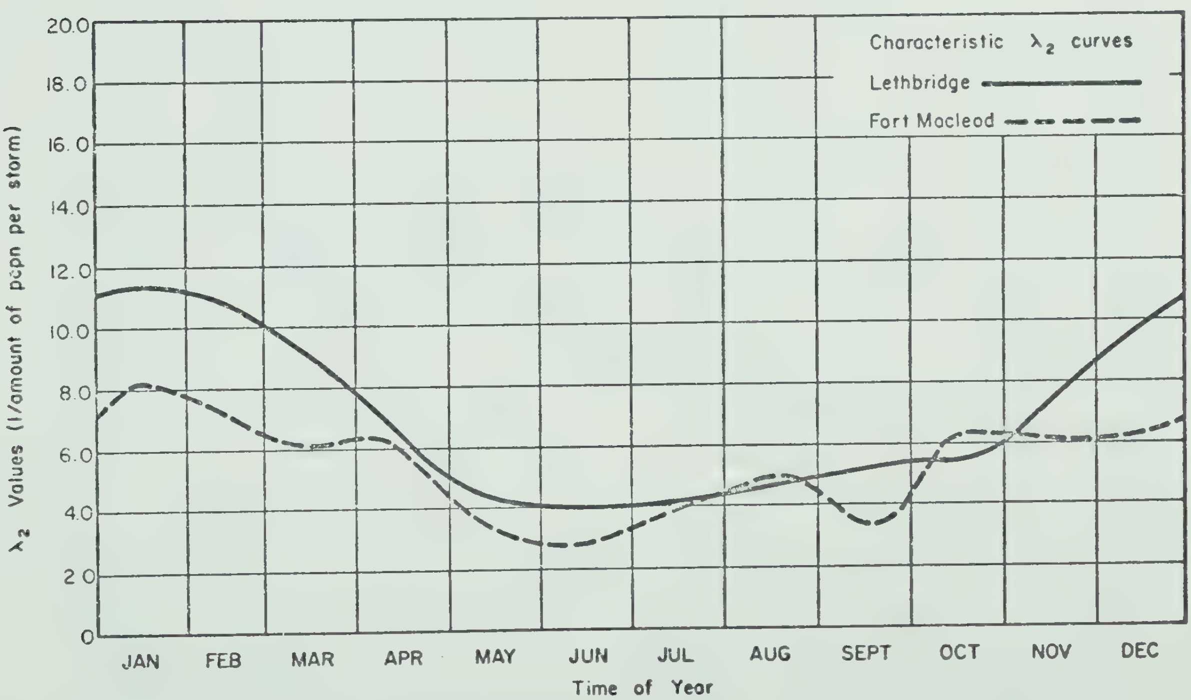

Fig. 51. Comparison of the characteristic λ_2 curves for Lethbridge and Fort Macleod

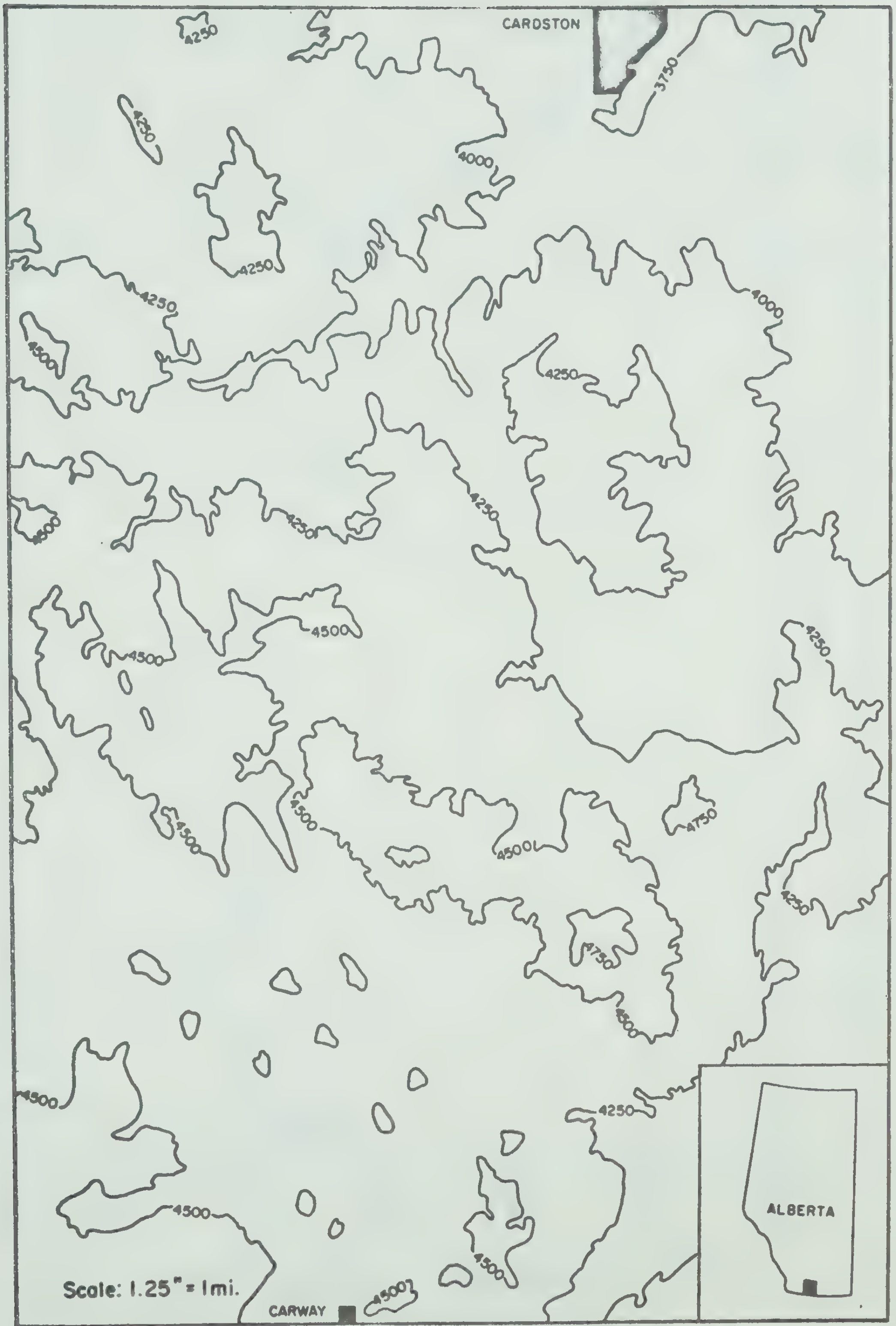

Fig.52. Contour map of the Cardston – Carway region

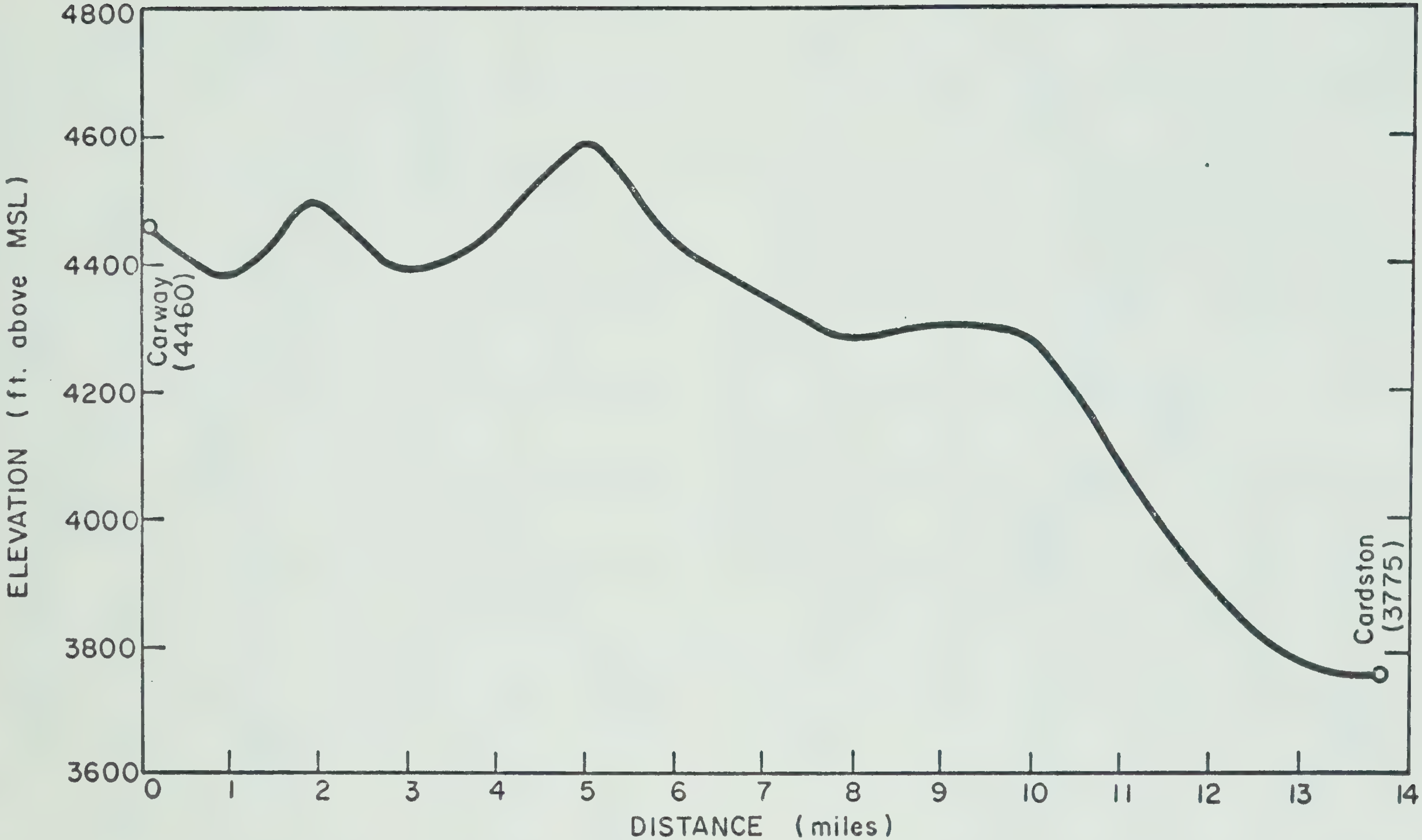

Fig. 53. Elevation profile of the Cardston–Carway region

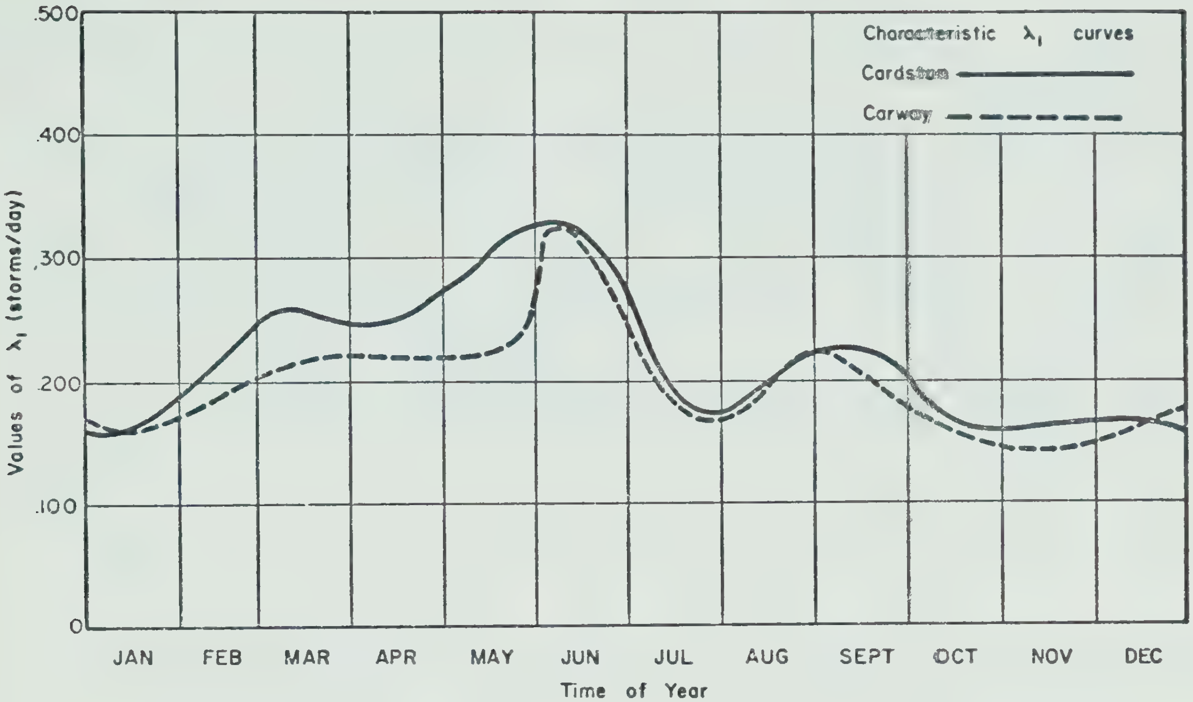

Fig. 54. Comparison of the characteristic λ_1 curves for Cardston and Carway

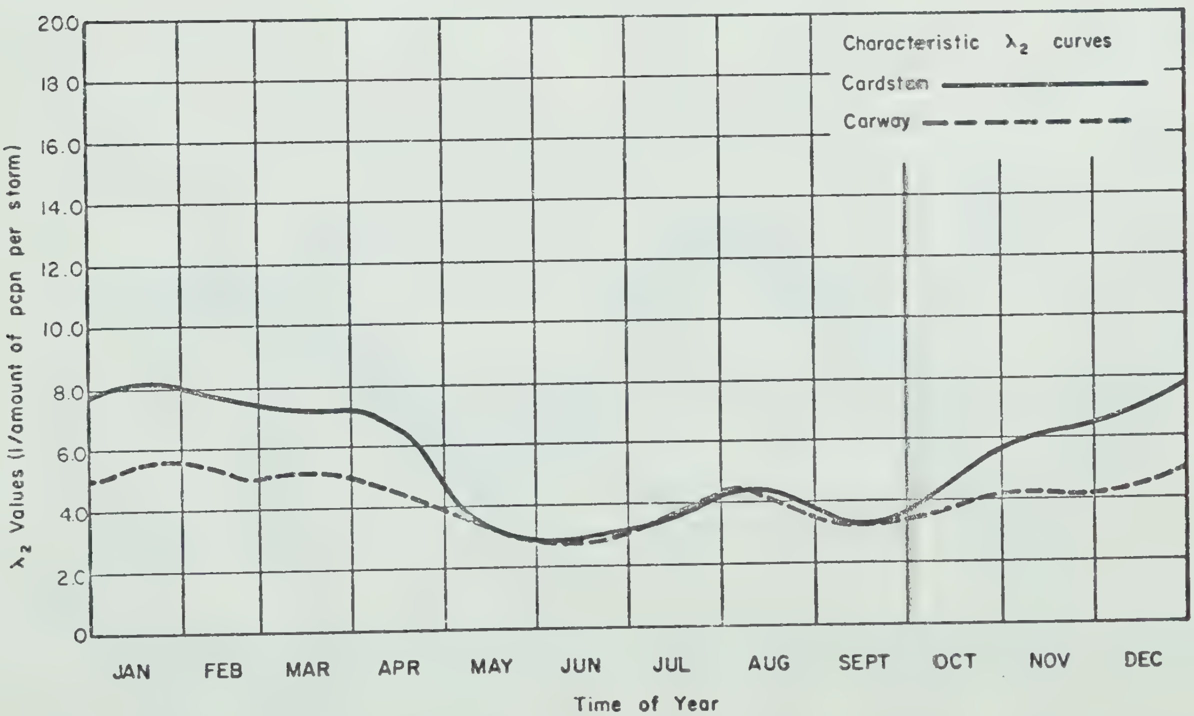

Fig. 55. Comparison of the characteristic λ_2 curves for Cardston and Carway

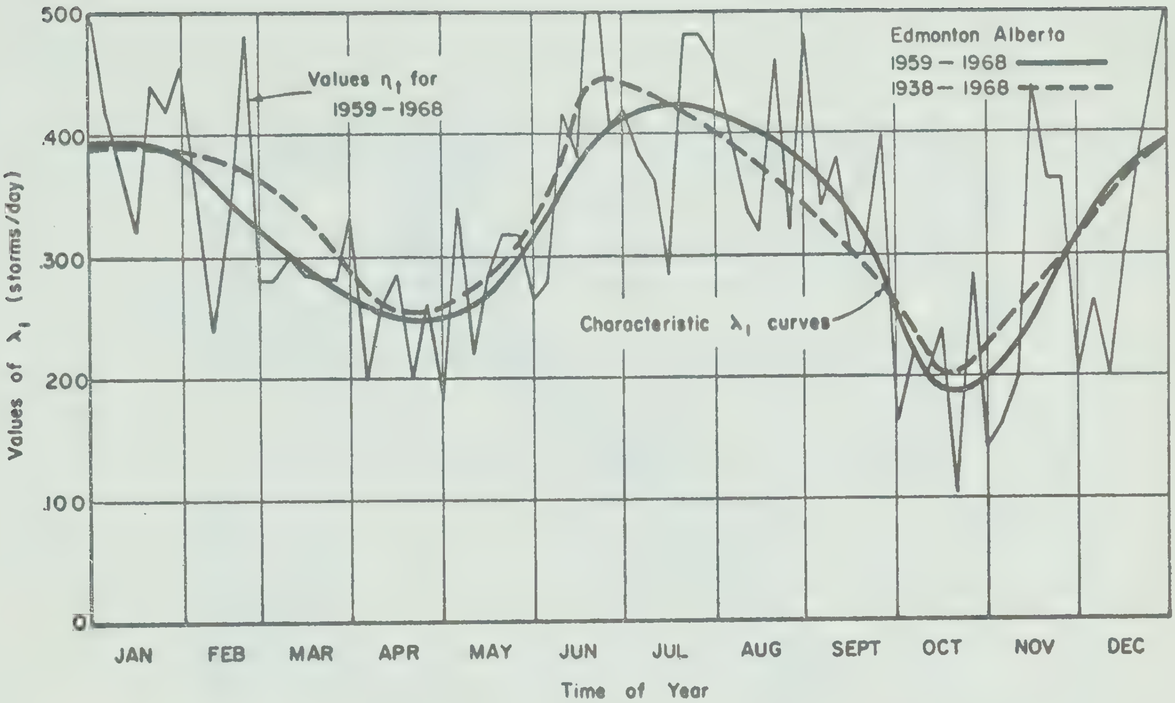

Fig. 56 Edmonton - characteristic λ_1 curves for 10 and 31 year record lengths

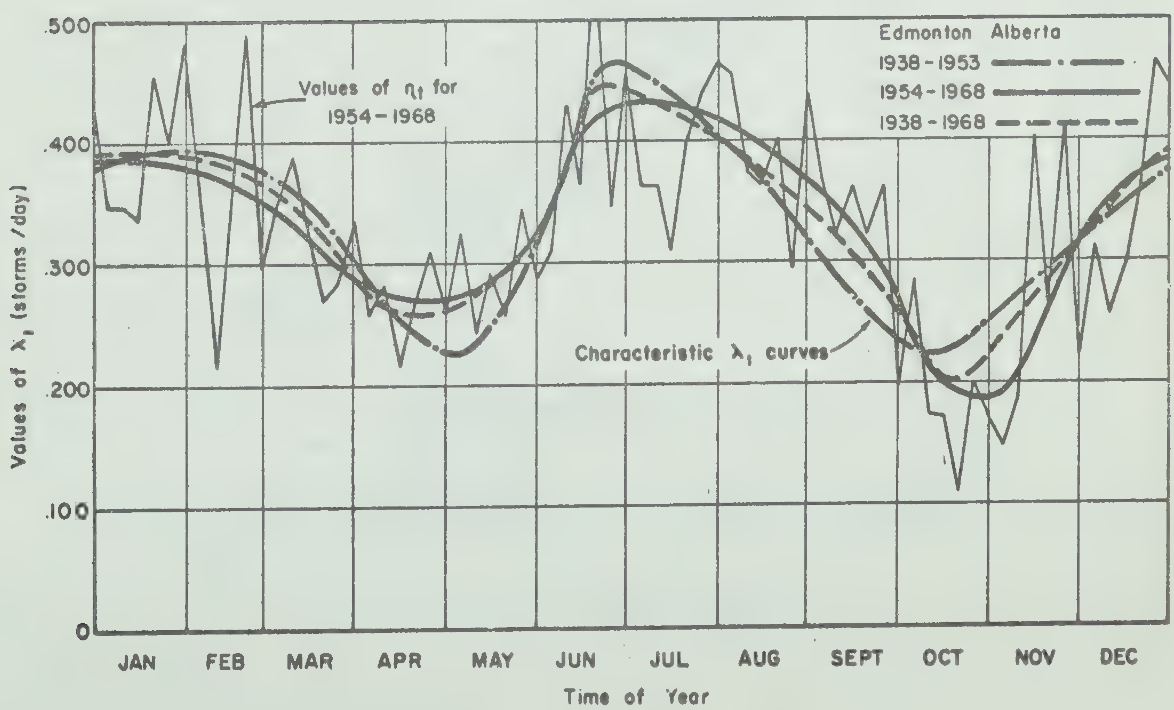

Fig. 57. Edmonton - characteristic λ_1 curves for 15 and 31 year record lengths

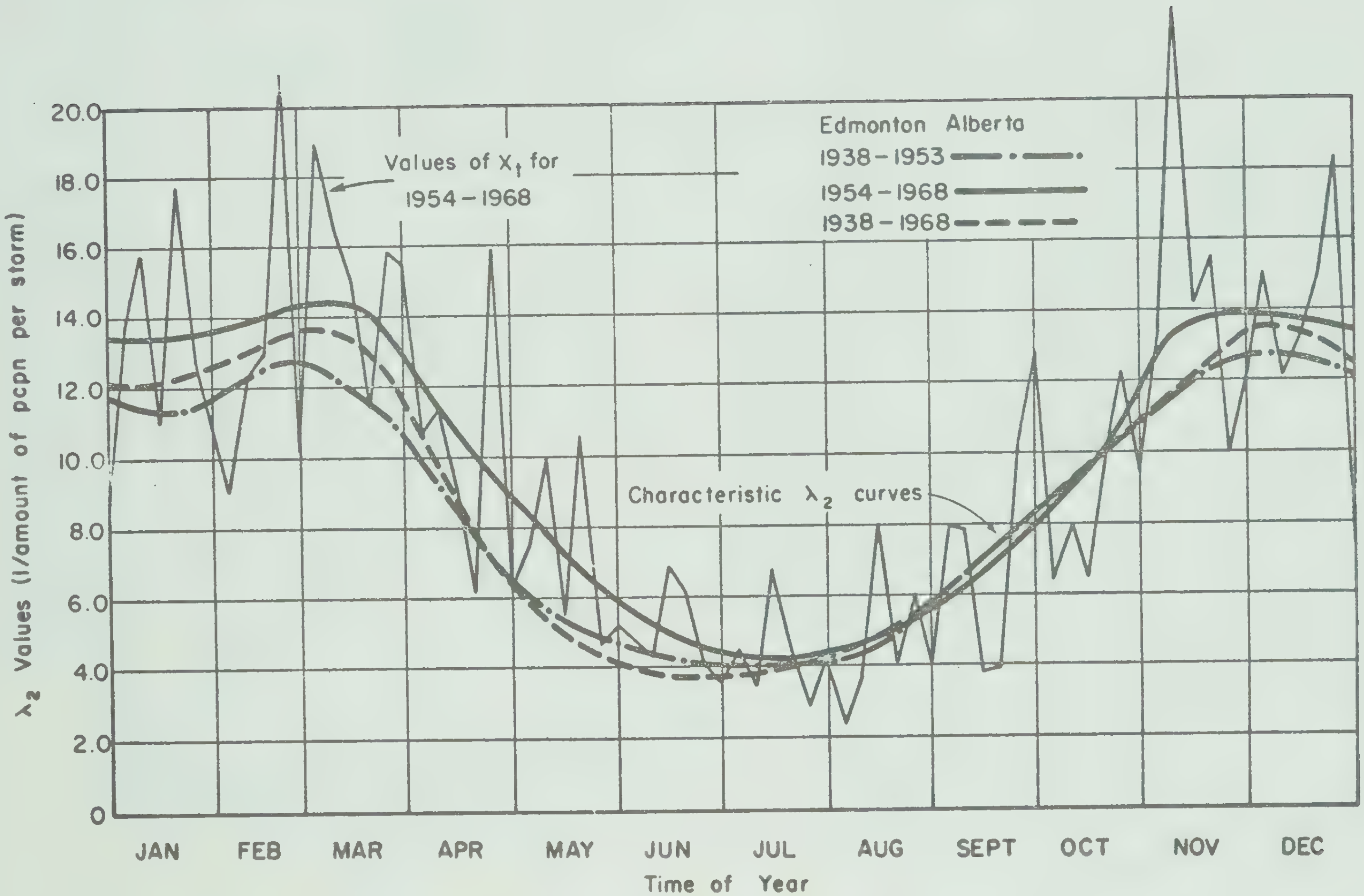

Fig. 58. Edmonton - characteristic λ_2 curves for 15 and 31 year record lengths

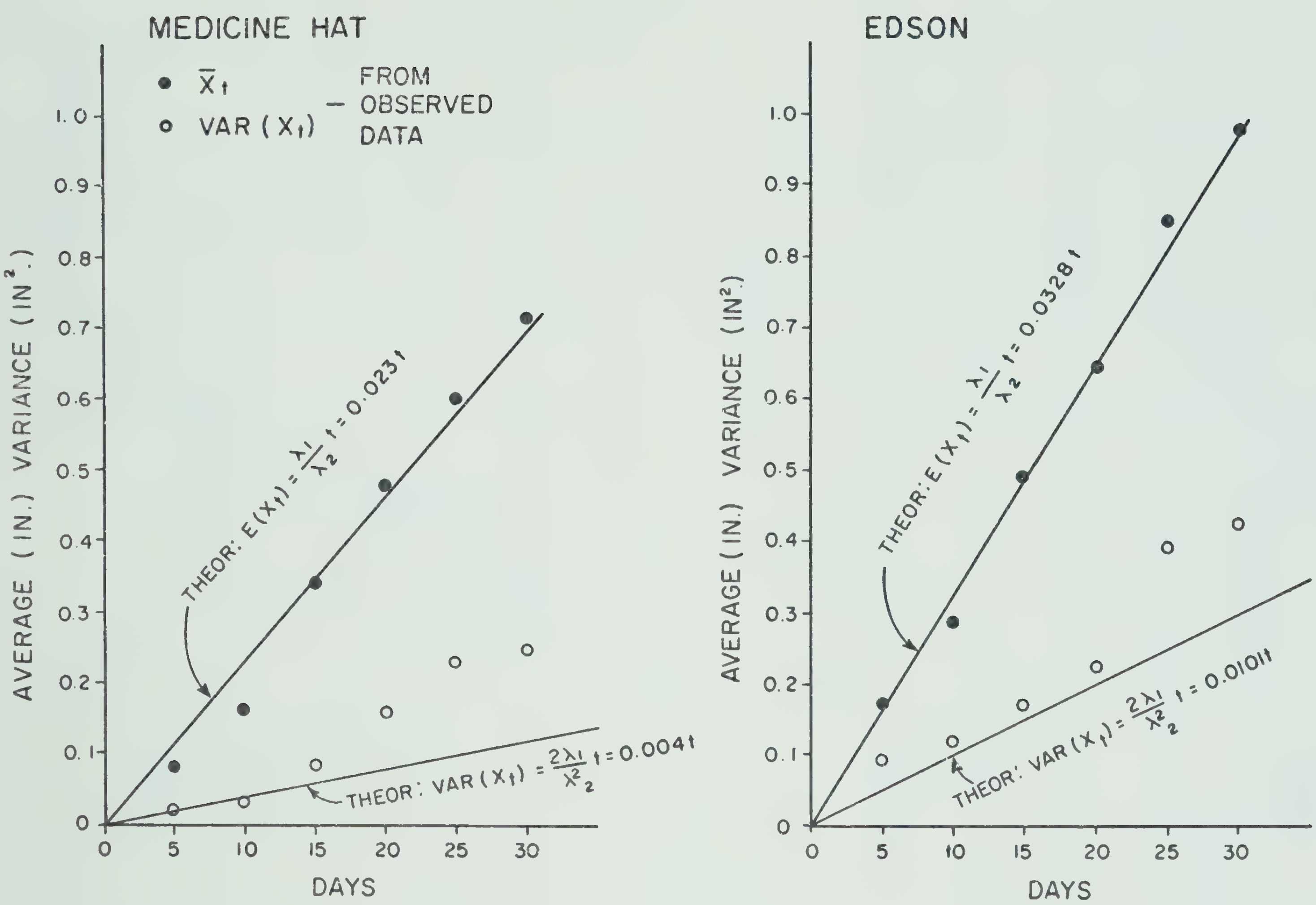

Fig.59. Observed and theoretical distributions of $E(X_t)$ and $Var(X_t)$

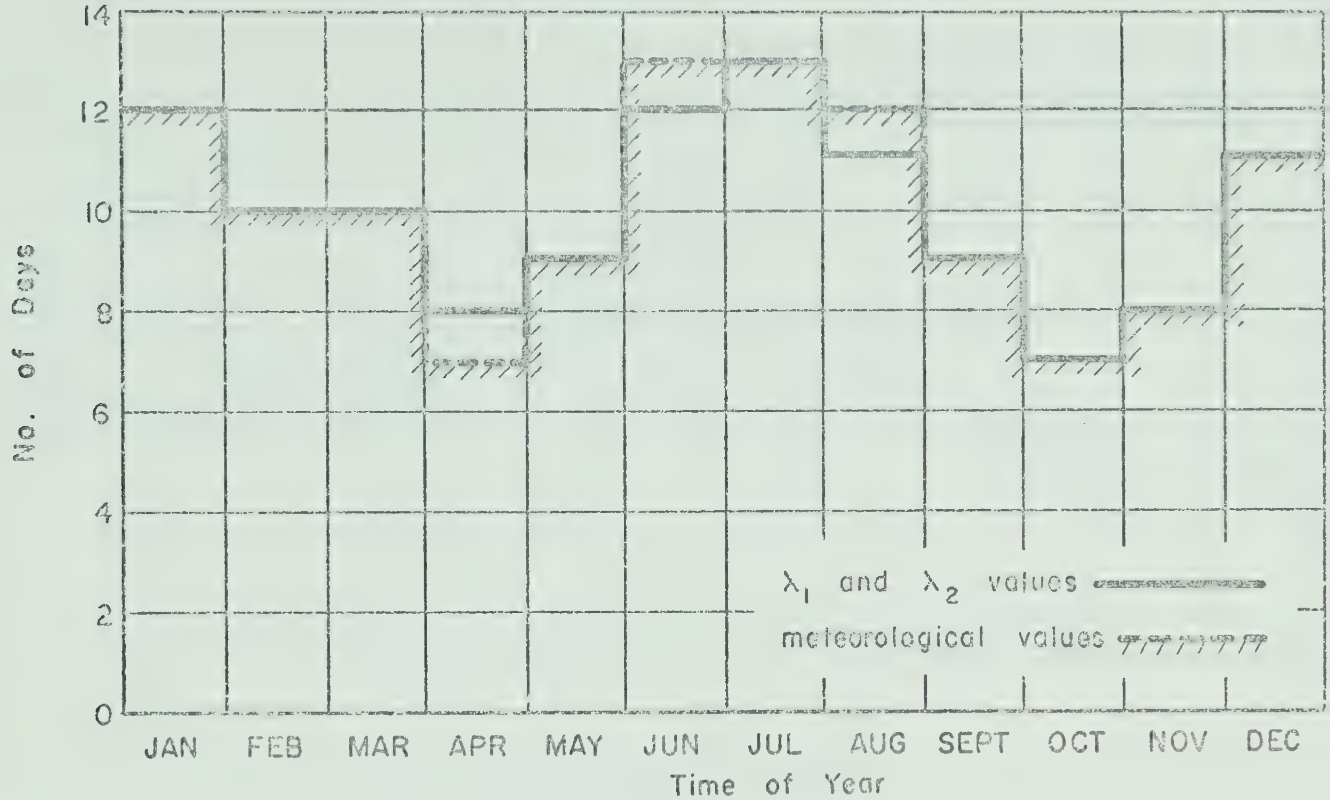

Fig. 60 Edmonton — Average monthly no. of days with measureable precipitation

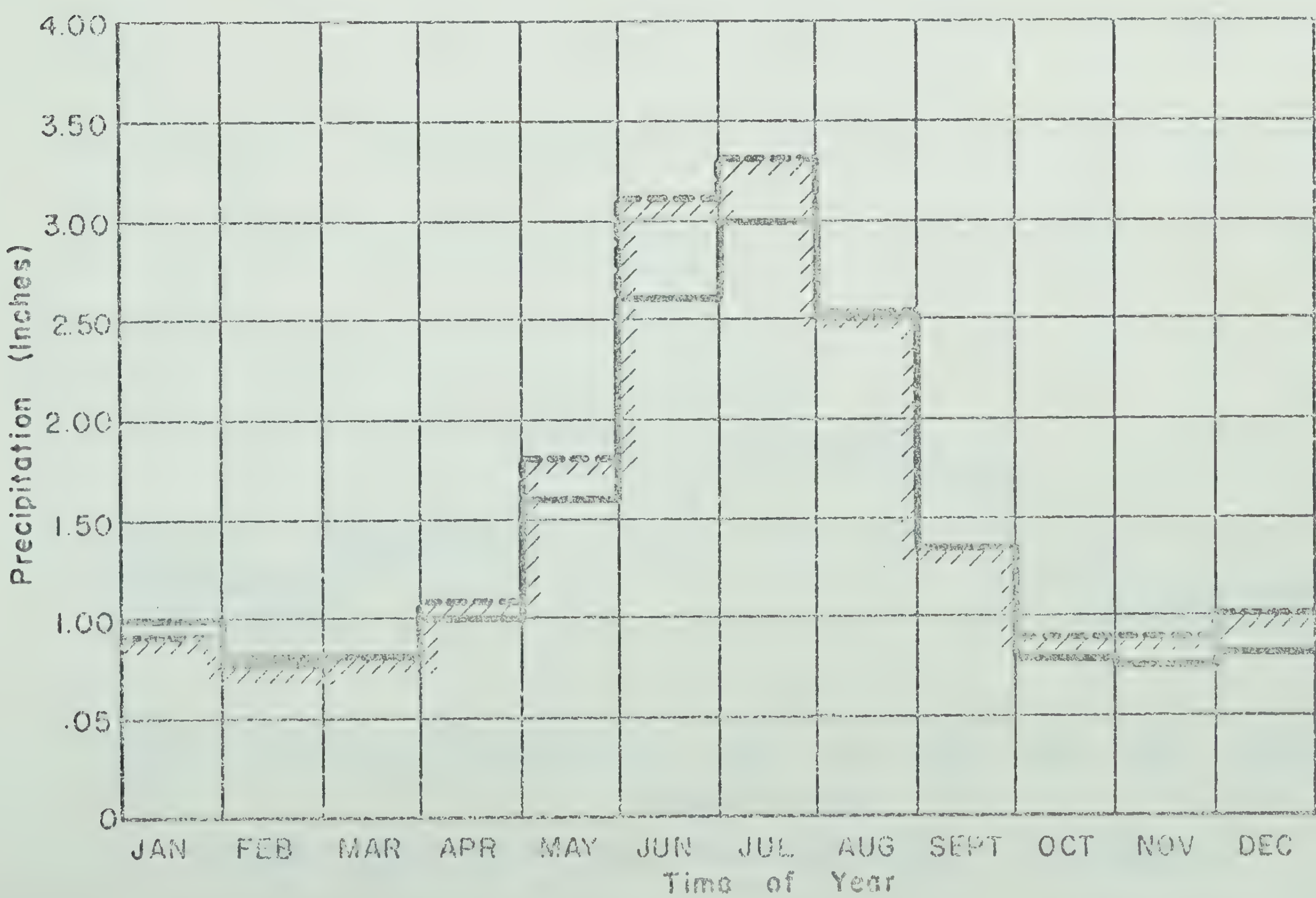

Fig. 61 Edmonton — Average monthly amount of precipitation

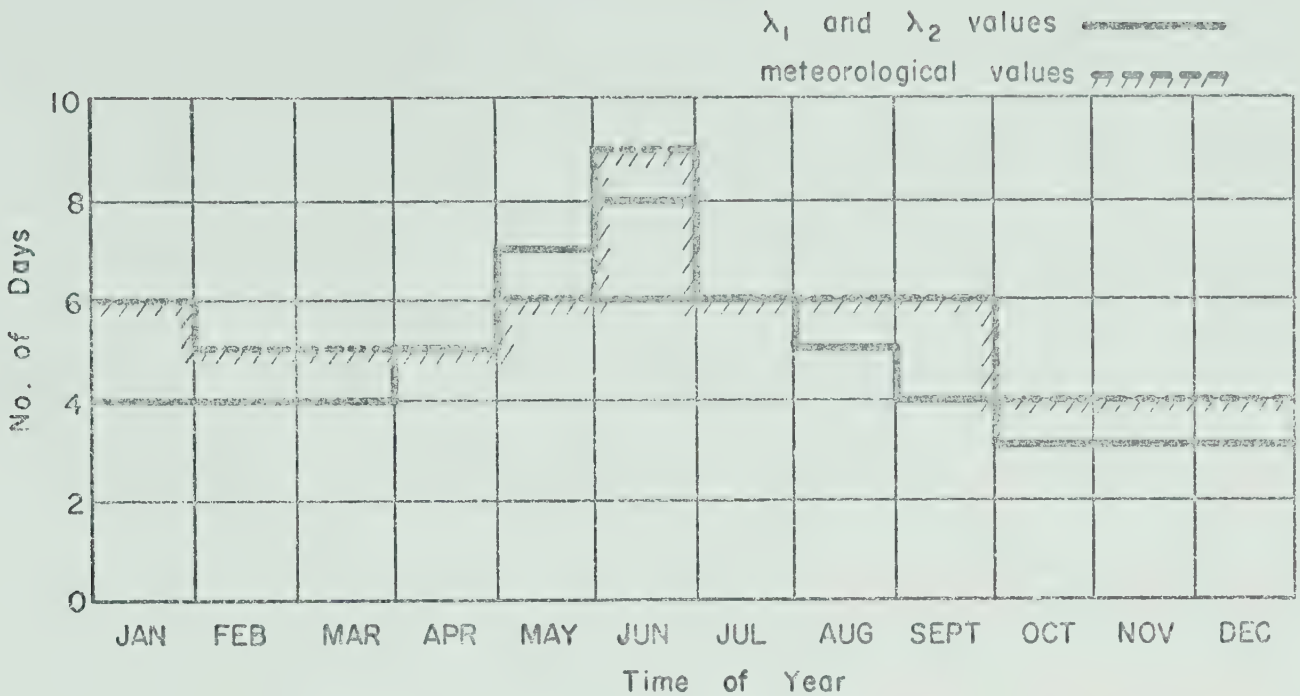

Fig. 62 Seven Persons – Average monthly no. of days with measureable precipitation

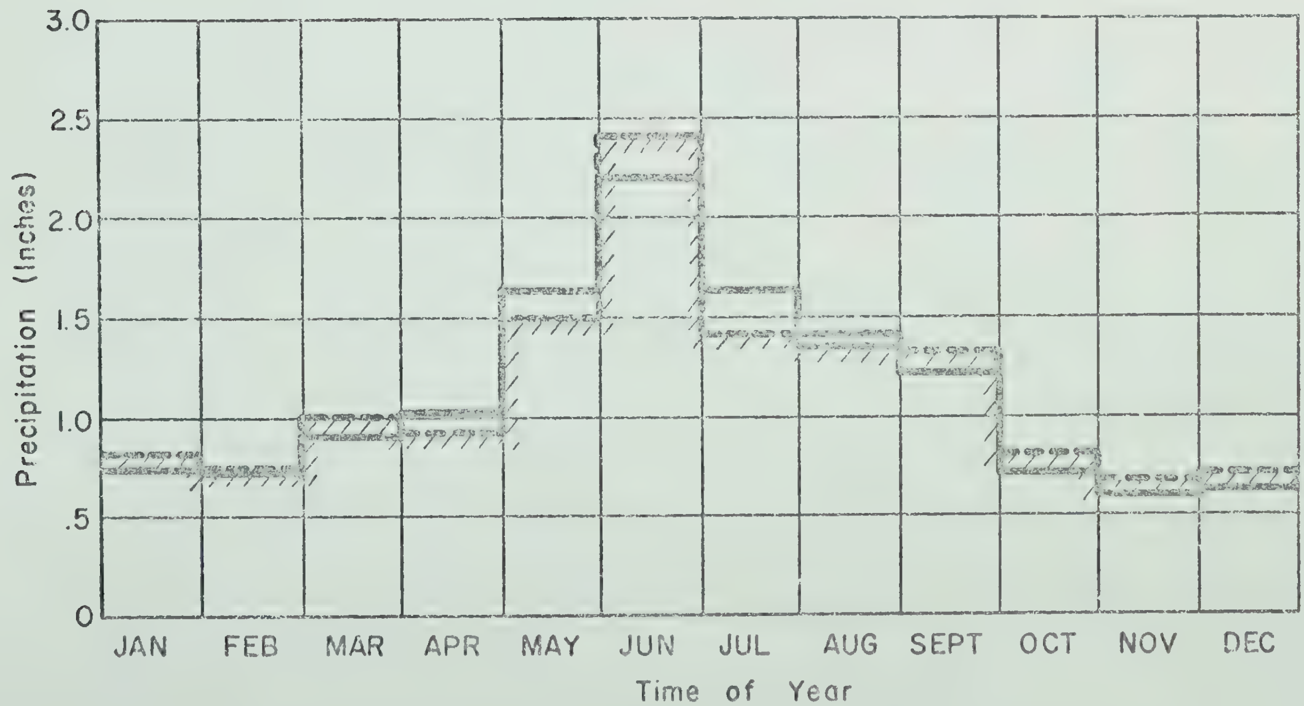

Fig. 63 Seven Person – Average monthly amount of precipitation